Binary System

Today’s numerical systems are based in the positional notation: the location of each digit in a string determines its “weight”. In the case of the decimal notation, the number 385, for example, is just an abbreviation for the number 300+80+5. The weights of the digits 5, 8 and 3 are the successive potencies of 10, i.e., 1, 10, and 100.

Binary notation uses the same positional representation, but the weights of the numbers are potencies of 2, which is called the base of the system. In base 2, only the digits 0 and 1 are used at each position. The number 1001, in binary, corresponds therefore to the number 8+1=9 in decimal notation. The weights of the two ones in the number 1001 are 8 and 1 respectively.

The German philosopher Gottfried Wilhelm Leibniz was the first mathematician who thoroughly studied the properties of the binary system. In several papers and letters, stretching from 1679 until 1697, Leibniz developed a notation for binary numbers and showed how to perform the basic arithmetic operations. The binary system was important for the metaphysicist Leibniz because “the void and obscurity correspond to zero and nothing, but God’s radiant spirit corresponds to the one”. The binary system was also known to the Chinese and the hexagrams of the *I Ching*, the “book of changes”, contain some binary pictorial representations. The Buddhist doctrine of Ying and Yang operated with these two “binary” principles from which the world could be constructed. Leibniz got acquainted with the *I Ching* hexagrams through a letter exchange with a missionary who stayed in Peking around 1700.

Numbers can be transformed from decimal to binary notation by a process of successive division. To transform the number 123 from decimal to binary, the number is iteratively divided by 2 and the remainder of the division is stored. The remainders, from the last to the first, written from left to right, provide the binary representation of the number. In the example below, we write the quotient of the division under the number being divided, and the remainder to the left, separated by a line:

123	
61	1
30	1
15	0
7	1
3	1
1	1
0	1

The iterative process ends when the quotient is 0. The binary representation of 123 is therefore 1111011.

Transforming numbers from binary to decimal is simpler: just multiply each binary digit by its weight, performing the operations in decimal. The binary number 111, for example, corresponds to the number 4+2+1=7.

Computers use the binary system because it is simpler to build logic gates that deal with only two states instead of 10, i.e. the number of states needed for each decimal digit. The hardware that performs additions of binary numbers, for example, has to deal only with the four cases 0+0, 0+1, 1+0, and 1+1. A decimal addition unit has to process many more combinations of digits. Multiplication is also very simple in the binary system: only the multiplication table for the digits 0 and 1 has to be “learned”. The multiplication table for the decimal system is much harder to remember, as the reader knows from his own experience.

Two early computers in the USA used a mixed internal representation: the ENIAC worked with decimal arithmetic, but each decimal digit was represented by ten lamps in a row. If, say, the fifth lamp was off, and all others on, the number stored was a five. The Harvard Mark I used only decimal storage in the form of rotating dials that could stop at one of ten positions, like in desktop calculators used in the 1940s.

Although computers compute internally using binary numbers, programmers usually abbreviate the numbers using octal or hexadecimal notation, that is, base 8 or 16. A number can be transformed from binary to octal notation just by grouping successive sets of three binary digits: the binary number 111101, for example, corresponds to the octal number 75. The reader can readily check that 111 is the binary representation of 7, 101 the binary representation of 5, and that 75 in base 8 is the same decimal number as 111101 in base 2. Octal numbers are written using the 8 digits from 0 to 7. Hexadecimal notation requires more digits, sixteen to be exact. The digits 0 to 9, followed by the letters A, B, C, D, E , and F are used for this purpose. The hexadecimal number A7 corresponds to the decimal number 10×16+7=167. Transforming binary numbers to hexadecimal notation is done like in the case of octal, but groups of four binary digits are used. The binary number 00110111, for example, corresponds to the hexadecimal number 37H. The trailing H is used to mark the number as an hexadecimal number. In the case of octal numbers, a trailing O is used.

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