9th Problem Set for the MDS Block Course on

High-Dimensional Computational Geometry

Helmut Alt, Wolfgang Mulzer Summer 2013

The problems will be discussed in the afternoon session.

Problem 1 Metric spaces

Which of the following pairs (X, ρ) are metric spaces:

(a) $X = \mathbb{R}^d, \rho(x, y) = \|x - y\|_2$

(b) $X = \mathbb{R}^d, \rho(x, y) = \|x - y\|_2^2$

(c) X is the set of continuous parameterized curves $f : [0, 1] \to \mathbb{R}^d$, ρ is the Fréchet distance.

(d) $X = \Sigma^*$, i.e., the set of all strings over a finite alphabet Σ, $\rho(x, y)$ is the edit distance between strings x and y, i.e., the minimum number of insertions, deletions, or replacements of characters necessary to transform x into y.

Problem 2 Embeddings

(a) Can every metric space of three elements be isometrically embedded into ℓ_2^2?

(b) Show that the two examples of 4-point metric spaces (square and star) shown in the course cannot be isometrically embedded into ℓ_2^d for any $d \in \mathbb{N}$.

(c) What is their minimum distortion when embedded into ℓ_2^2?

Problem 3 Johnson-Lindenstrauss

In the proof of the Johnson-Lindenstrauss Theorem we chose $k = 200\varepsilon^{-2}\ln(n)$.

(a) Replace 200 by the smallest constant that is possible if $n \geq 10$, say.

(b) Find a constant such that the probability, that the projection into a randomly chosen k-dimensional subspace is a $(1 + \varepsilon)$-embedding is greater than $1 - 1/n$. Which constant is necessary for the bound $1 - 1/n^r$?