8th Problem Set for the MDS Block Course on

High-Dimensional Computational Geometry

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The problems will be discussed in the afternoon session.

Problem 1 Near-Neighbor Queries I

Fix $a > 1$. Suppose we have a data structure for c-approximate 1-near neighbor queries under the assumption that the points lie in the cube $[0, a]^d$. The distance for the queries is measured in the ℓ_1 (Manhattan) metric. Let f be the failure probability of the data structure.

Show that then we can also construct a structure for c-approximate 1-near neighbor structure for points from $\mathbb{R}^d$ (for the ℓ_1 metric). The resulting data structure should have asymptotically the same parameters and failure probability $f + 1/a$.

Hint: Impose a randomly shifted grid of appropriate size over $\mathbb{R}^d$ and bound the probability that two points of ℓ_1-distance at most 1 lie in different grid cells. Build the given data structure for each non-empty grid cell.

Problem 2 Near-Neighbor Queries II

Let $a > 1$ and $\delta > 0$. Set $M = ad/\delta$. Suppose we have a c-approximate r-near neighbor structure for the dM-dimensional Hamming cube with respect to the Hamming metric.

Show that then we can construct a $(1 + \delta)c$-approximate 1-near neighbor structure for points from $[0, a]^d$ in the ℓ_1-metric.

Hint: First, round all points to a multiple of δ/d. Then consider the function $u : \{0, \ldots, M\} \to \{0, 1\}^M$ that maps a number m to a string of m zeros followed by $M - m$ ones. Show that after applying u to each coordinate, the Hamming distance corresponds to the ℓ_1 distance.

Problem 3 Separating hyperplanes

Suppose two finite sets of points in $\mathbb{R}^d$ are given, A ("red points") and B ("blue points"). For a given hyperplane h separating $\mathbb{R}^d$ into halfspaces h^+ and h^-, define the *imbalance* of h as $min(|A \cap h^+| + |B \cap h^-|, |A \cap h^-| + |B \cap h^+|)$.

Design and analyze algorithms which for input sets A and B:

(a) determine whether there exists a separating hyperplane, i.e., one with imbalance 0,

(b) find a hyperplane minimizing the imbalance.