7th Problem Set for the MDS Block Course on

High-Dimensional Computational Geometry

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The problems will be discussed in the afternoon session.

Problem 1 Reduction to the Near-Neighbor Problem

Fill in the missing details in the reduction from the nearest-neighbor to the near-neighbor problem. In particular, give the details of the approximate clustering algorithm, analyze the space requirement, and show that the choice of parameters give the desired approximation.

Problem 2 A Simple Near-Neighbor Structure

Let $\varepsilon \in (0, 1/2)$ and set $c = 1 + \varepsilon$. Show how to solve the c-approximate r-near neighbor problem in d dimensions with $n \cdot O(1/\varepsilon)^d$ preprocessing and space, and query time $O(d)$. You may use hashing and the floor function. How do you feel about this result?

Hint: Use a grid of appropriate size. It may be useful to know that the volume of the d-dimensional ball with radius a is

$$\frac{\pi^{d/2}}{(d/2)!} \cdot a^d,$$

for d even and

$$\frac{2(2\pi)^{(d-1)/2}}{d!!} \cdot a^d,$$

for d odd. Here, $d!!$ is the *double factorial*, defined as $d!! = \prod_{i=0}^{(d-1)/2}(2i+1)$, the product of all odd integers from 1 to d.

Problem 3 The Number of Resolutions

Let $P \subseteq \mathbb{R}^d$ with $|P| = n$. Set

$$U = \{i \in \mathbb{Z} \mid \text{there are } p, q \in P \text{ with } 2^i \leq d(p,q) < 2^{i+1}\}.$$

Show that $|U| = O(n)$.

Hint: One approach might be to consider a compressed quadtree for P and to charge the resolutions to the nodes of the quadtree.