6th Problem Set for the MDS Block Course on

High-Dimensional Computational Geometry

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The problems will be discussed in the afternoon session.

Problem 1 Approximating r_{med}

Let P be a d-dimensional n-point set. Define r_{med} as the minimum radius such that $\text{CC}_P(r_{\text{med}})$ contains a component with at least $n/2 + 1$ points.

Consider the following algorithm: Take a random point $p \in P$ and let r^*_{med} be the median of the distances $d(p, q)$ for $q \in P \setminus \{p\}$. Show that with probability at least $1/2$, we have $r_{\text{med}} \leq r^*_{\text{med}} \leq 2(n-1)r_{\text{med}}$.

Problem 2 Voronoi Diagram in $\mathbb{R}^3$

Show that the three-dimensional Voronoi diagram for n points can have complexity $\Omega(n^2)$.

Problem 3 Exact Nearest Neighbor II

What is the best query time that you can achieve for the exact nearest neighbor problem if the data structure is allowed space $O(n^{\lceil d/2 \rceil})$?

Hint: One possible strategy would be to adapt the randomized convex hull algorithm.