Sandwich Approximation of Univariate Convex Functions with an Application to Separable Convex Programming

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In this article an algorithm for computing upper and lower ε approximations of a (implicitly or explicitly) given convex function h defined on an interval of length T is developed. The approximations can be obtained under weak assumptions on h (in particular, no differentiability), and the error decreases quadratically with the number of iterations. To reach an absolute accuracy of ε the number of iterations is bounded by $\sqrt{9DT/8\varepsilon}$, where D is the total increase in slope of h. As an application we discuss separable convex programs.

1. INTRODUCTION

Convex functions play an important role in mathematical programming. Many models lead directly to convex functions, or they arise as value functions of parametric linear programs, in time/cost trade-off problems, or in multicriteria optimization. There are several reasons for replacing a convex function by a piecewise linear approximation with few breakpoints: In some models piecewise linear functions are easier to handle, for example, in separable convex programming. This will be utilized in Section 3. Another reason is that the evaluation of a convex function $h(t)$ for a given parameter t may be costly. Therefore one is interested in getting an approximation of this function with as few function evaluations as possible. This arises, for instance, if $h(t)$ is the value function of a parametric linear program: Evaluating $h(t)$ amounts to solving a linear program. The same situation occurs in the context of bicriteria linear programs, since the efficient point curve is convex. The special case of bicriterial minimum

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