Algorithms for wireless networks
Problem Set 1

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Due in class on December 13, 2012

Problem 1 (10 points)

A Gabriel graph (GG) for a set of vertices in the plane is defined as follows. The edge (v_i, v_j) between vertices v_i and v_j is in GG if the disk with the segment $v_i v_j$ as diameter does not contain any other vertices. In class, we have seen that GG is planar.

Show that the gabriel graph contains the minimum spanning tree (MST). That is, any edge of the minimum spanning tree is always an edge of the gabriel graph.

Problem 2 (10 points)

For a set of vertices in the plane, the Euclidean stretch of a path P between vertices v_i and v_j is the ratio of the length of P to the Euclidean distance $v_i v_j$. The Euclidean spanning ratio of a graph is the largest stretch over all pairs of vertices taken with respect to the shortest path between them in the graph.

(a) Show that there are sets of vertices for which the Euclidean spanning ratio of the minimum spanning tree is not a constant.

(b) Similarly, show that the spanning ratio of a GG may be larger than any constant.

(problem 3 continued on next page)

Problem 3 (5 points)

In creating MST, the length of each edge is called *weight*, and MST is the tree with least total weight.

if we add an edge between two vertices of MST that are not adjacent in the MST, then this creates a cycle. Show that a MST has the property of cycle optimality. That is, the weight of the newly added edge cannot be smaller than the weight of any of the edges in the cycle it creates.

Problem 4 (15 points)

The construction of MST of vertices in the plane needs to measure the distances between pairs of points. Now suppose our measurement system has errors. For example, these are points in a large field and we are making measurements using imprecise instruments.

We assume that the relative error is bounded. That is, there is a global constant $\delta > 1$ such that our measurement of a segment of length ℓ returns a value between $\frac{\ell}{\delta}$ and $\ell \cdot \delta$.

Show that the total weight of the MST computed with these measurements is within a factor of δ of the weight of the true MST.