Math. für Informatiker III – Nachklausur vom 7.4.2009

1.

$$\operatorname{rg}\begin{pmatrix}1 & 2 & 6\\ 4 & 1 & 4\\ 6 & 5 & z-6\end{pmatrix} \begin{matrix} \\ -4 \\ -6\end{matrix} = \operatorname{rg}\begin{pmatrix}1 & 2 & 6\\ 0 & -7 & -20\\ 0 & -7 & z-42\end{pmatrix} \begin{matrix} \\ \\ -\end{matrix} = \operatorname{rg}\begin{pmatrix}1 & 2 & 6\\ 0 & -7 & -20\\ 0 & 0 & z-22\end{pmatrix}$$

$\operatorname{rg} A = 3$ für $z-22 \neq 0$, $z \neq 22$: Für $z \neq 22$ gibt es <u>eine eindeutige Lösung</u>.

$z=22$: $\operatorname{rg} A = 2$

$$\operatorname{rg}(A\,{:}\,b) = \operatorname{rg}\left(\begin{array}{ccc|c}1 & 2 & 6 & 5\\ 4 & 1 & 4 & 0\\ 6 & 5 & 16 & -1\end{array}\right) = \operatorname{rg}\left(\begin{array}{ccc|c}1 & 2 & 6 & 5\\ 0 & -7 & -20 & -20\\ 0 & -7 & -20 & -31\end{array}\right) = \operatorname{rg}\begin{pmatrix}1 & 2 & 6 & 5\\ 0 & -7 & -20 & -20\\ 0 & 0 & 0 & -11\end{pmatrix} = 3$$

$\operatorname{rg} A < \operatorname{rg}(A\,{:}\,b) \Rightarrow$ Für $z=22$ hat das System <u>keine Lösung</u>.

2. $2^3 = \underline{\underline{8}}$ Codewörter. Prüfmatrix $H = \left(\begin{array}{ccc|cc}1 & 1 & 0 & 1 & 0\\ 1 & 0 & 1 & 0 & 1\end{array}\right)$. Informationsrate $= \frac{\log_2 8}{5} = \underline{\underline{3/5}}$.

3.
a) falsch: z.B. $A = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$, $B = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$, $C = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $(A\cdot B\cdot C)^t = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$ $A^t\cdot B^t\cdot C^t = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$

b) wahr: $(A+B)^t = A^t + B^t$ (Summe von transpon. Matrizen) $= B^t + A^t$ (kommutativ)

c) falsch z.B. $A = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $\operatorname{rg} A + \operatorname{rg} A^t = 2+2 = 4 \neq \operatorname{rg}(A+A^t) = \operatorname{rg}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} = 2$

d) wahr: $(A+A^t)^t = A^t + (A^t)^t = A^t + A = A + A^t$ (kommutativ)

e) wahr: $S^t = (A-A^t)^t = A^t - (A^t)^t = A^t - A = -(A-A^t) = -S$

4.
a) $P(S \geq 10) = P(\{(6,4),(6,5),(6,6),(5,5),(5,6),(4,6)\}) = \frac{6}{36} = \underline{\underline{1/6}}$

b)
$(6,6) \xrightarrow{1/36} G = 12$
$(6,5),(5,6) \xrightarrow{2/36} G = 11$
$(1,1) \xrightarrow{1/36} G = 2$
$(1,2),(2,1) \xrightarrow{2/36} G = 3$
Sonst $\xrightarrow{30/36} G = 0$

$E(G) = \frac{1}{36}\cdot 12 + \frac{2}{36}\cdot 11 + \frac{1}{36}\cdot 2 + \frac{2}{36}\cdot 3 = \frac{42}{36} = \underline{\underline{7/6}}$

$\operatorname{Var}(G) = E(G^2) - (E(G))^2 = \left(\frac{1}{36}\cdot 144 + \frac{2}{36}\cdot 121 + \frac{1}{36}\cdot 4 + \frac{2}{36}\cdot 9\right) - \frac{49}{36} = \frac{359}{36} = 10 - \frac{1}{36} = \underline{\underline{9\tfrac{35}{36}}}$

5.

$u_1 = \begin{pmatrix}1\\4\\2\\2\end{pmatrix}$ $\|u_1\| = \sqrt{1+16+4+4} = \sqrt{25} = 5$ $\underline{\underline{v_1}} = \frac{u_1}{\|u_1\|} = \underline{\underline{\frac{1}{5}\begin{pmatrix}1\\4\\2\\2\end{pmatrix}}}$

$u_2 = \begin{pmatrix}-1\\5\\3\\0\end{pmatrix}$, $v_2' = u_2 - \langle u_2, v_1\rangle\cdot v_1 = u_2 - \underbrace{\frac{1}{5}(-1+20+6)\cdot\frac{1}{5}}_{1} u_1 = \begin{pmatrix}-1\\5\\3\\0\end{pmatrix} - \begin{pmatrix}1\\4\\2\\2\end{pmatrix} = \begin{pmatrix}-2\\1\\1\\-2\end{pmatrix}$

$\|v_2'\| = \sqrt{4+1+1+4} = \sqrt{10}$ $\underline{\underline{v_2}} = \frac{v_2'}{\|v_2'\|} = \underline{\underline{\frac{1}{\sqrt{10}}\cdot\begin{pmatrix}-2\\1\\1\\-2\end{pmatrix}}}$

$u_3 = \begin{pmatrix}-1\\-1\\2\\3\end{pmatrix}$, $v_3' = u_3 - \langle u_3, v_1\rangle\cdot v_1 - \langle u_3, v_2\rangle\cdot v_2 = u_3 - \frac{1}{5}\underbrace{(-1-4+4+6)}_{5}\cdot\frac{1}{5}u_1 - \frac{1}{\sqrt{10}}\underbrace{(2-1+2-6)}_{-3}\cdot\frac{1}{\sqrt{10}}v_2'$

$$v_3' = \begin{pmatrix}-1\\-1\\2\\3\end{pmatrix} - \frac{1}{5}\begin{pmatrix}1\\4\\2\\2\end{pmatrix} + \frac{3}{10}\begin{pmatrix}-2\\1\\1\\-2\end{pmatrix} = \begin{pmatrix}-1 - 2/10 - 6/10\\ -1 - 8/10 + 3/10\\ 2 - 4/10 + 3/10\\ 3 - 4/10 - 6/10\end{pmatrix} = \frac{1}{10}\begin{pmatrix}-18\\-15\\19\\20\end{pmatrix}$$

$\|v_3'\| = \frac{1}{10}\cdot\sqrt{324+225+381+400}$
$= \frac{1}{10}\cdot\sqrt{1310}$

$\underline{\underline{v_3}} = \frac{\|v_3'\|}{\|v_3'\|} = \underline{\underline{\frac{1}{\sqrt{1310}}\begin{pmatrix}-18\\-15\\19\\20\end{pmatrix}}}$. v_1, v_2, v_3 bilden eine Orthonormalbasis.

6.

$$\det\begin{pmatrix}1 & 1 & 1\\ -3 & -1 & 2\\ -2 & 1 & 5\end{pmatrix} = -5-4-3-2-2+15 = -1 < 0$$

$\Rightarrow$ Dreieck ist <u>im Uhrzeigersinn</u> orientiert