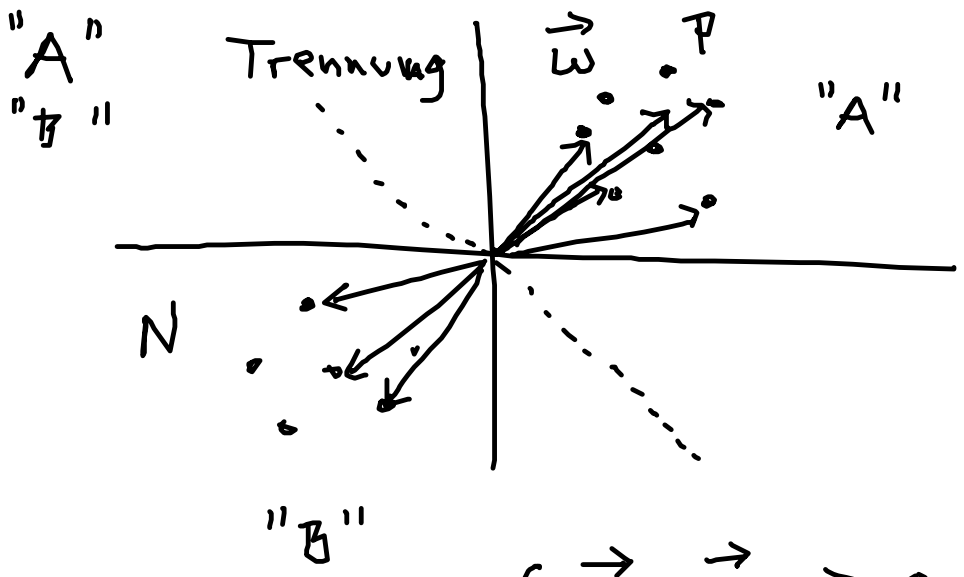
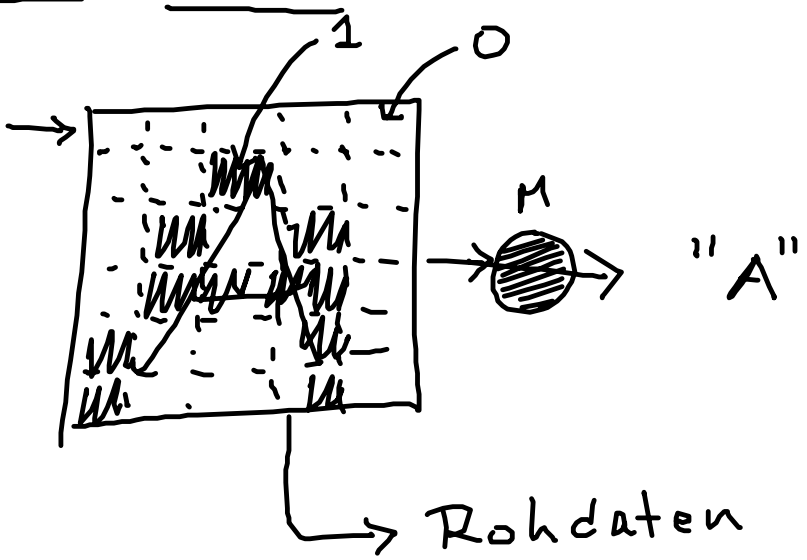
Mustererkennung

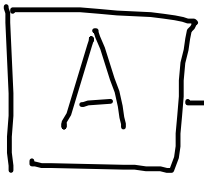


$$\begin{cases} \vec{p} \cdot \vec{\omega} \geq 0 \\ \vec{n} \cdot \vec{\omega} < 0 \end{cases}$$

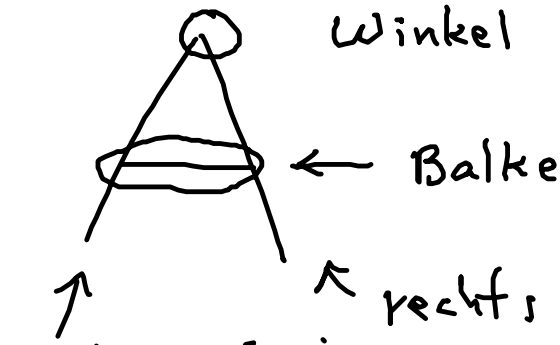
Perzepton

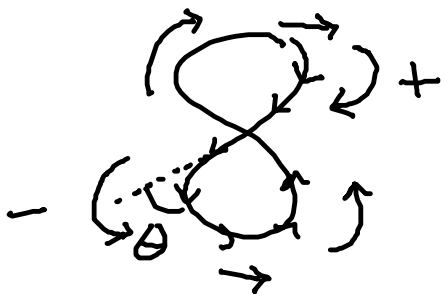
Brute Force





→ Merkmale





Merkmale

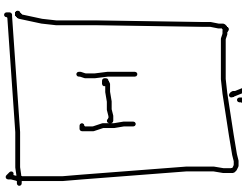
Linienzüge: 1

Kreuzungen: 1

kummulierte Drehung = 0

$\|\text{kum. Dreh}\| = 4\pi$

⋮

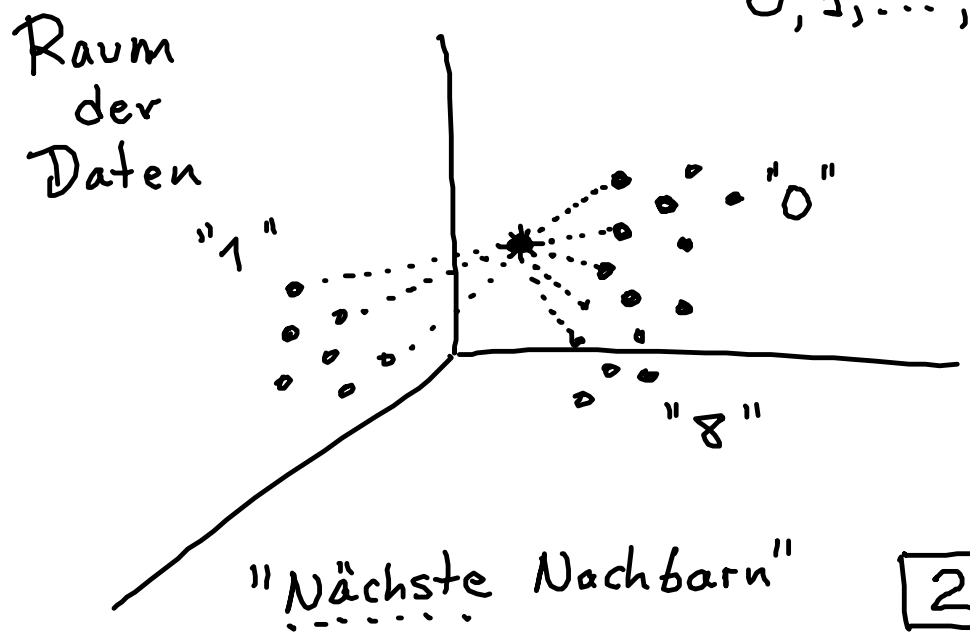


→ Rohdaten $\Longrightarrow \vec{x} \in \mathbb{R}^n$

→ Merkmale $\Longrightarrow \vec{x} \in \mathbb{R}^m$

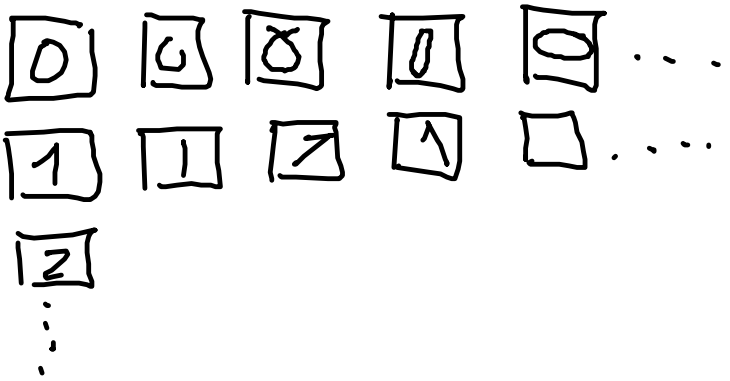
$m \ll n$

0, 1, ..., 9



"Nächste Nachbarn"

2 → Merkmale $\vec{x}$



$$\begin{cases} \text{Klasse von } \vec{x} = \text{Klasse vom nächsten Nachbarn} \\ \qquad = \text{k-"nächste" Nachbarn (Mehrheit)} \end{cases}$$

$K = 3$

k-nearest neighbor

"abgelehnt"

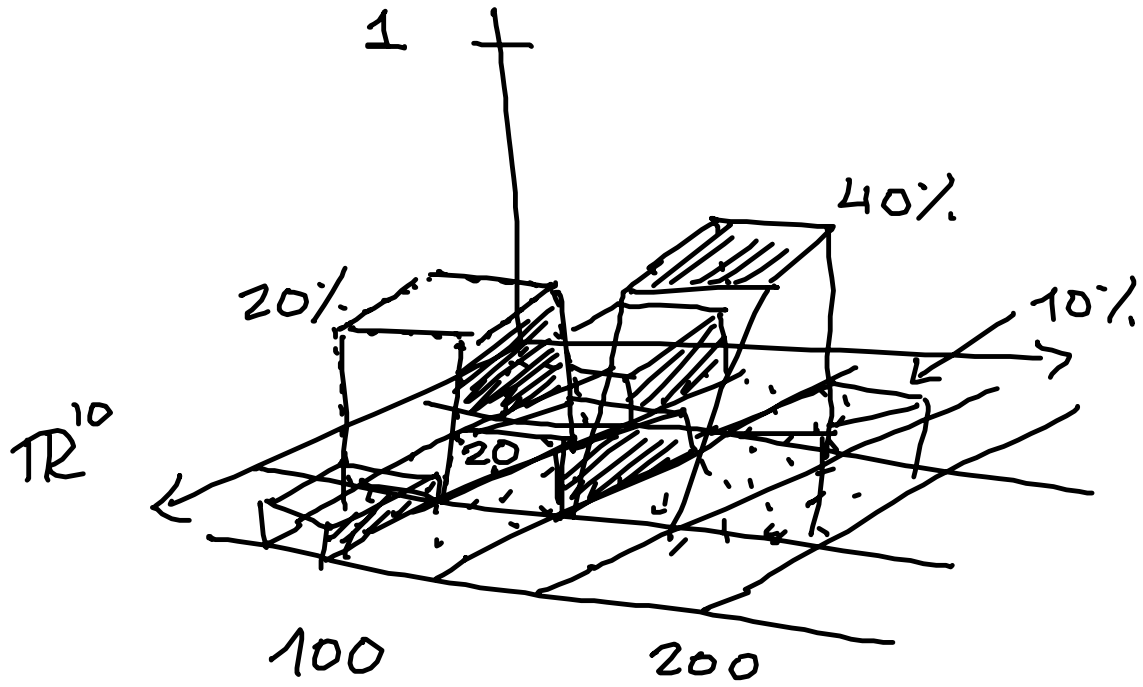
.2

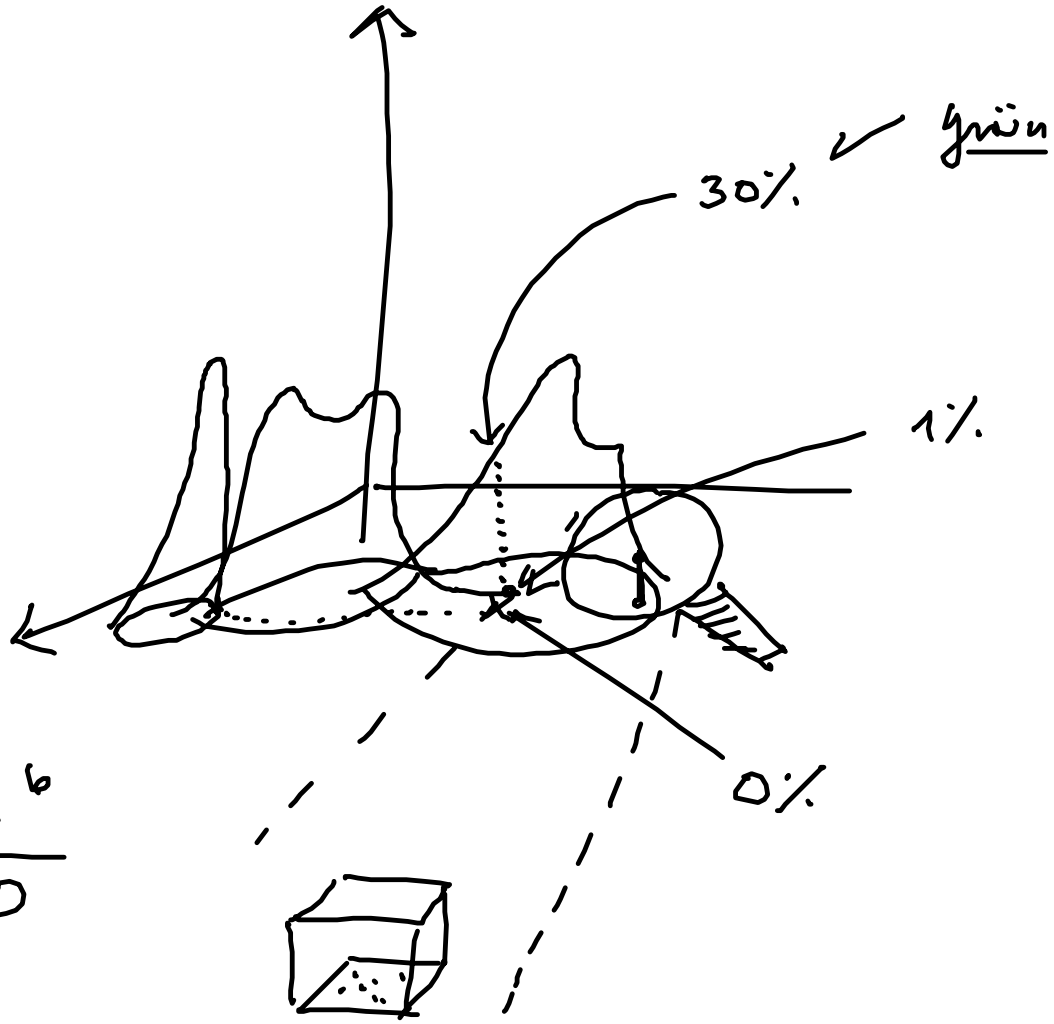
Nachteil: Laufzeit $\overbrace{10^6 \text{ Ziffern}}^{\text{Teuer}}$

$\mathbb{R}^{10}$

Einfacher (Rechenleistung)

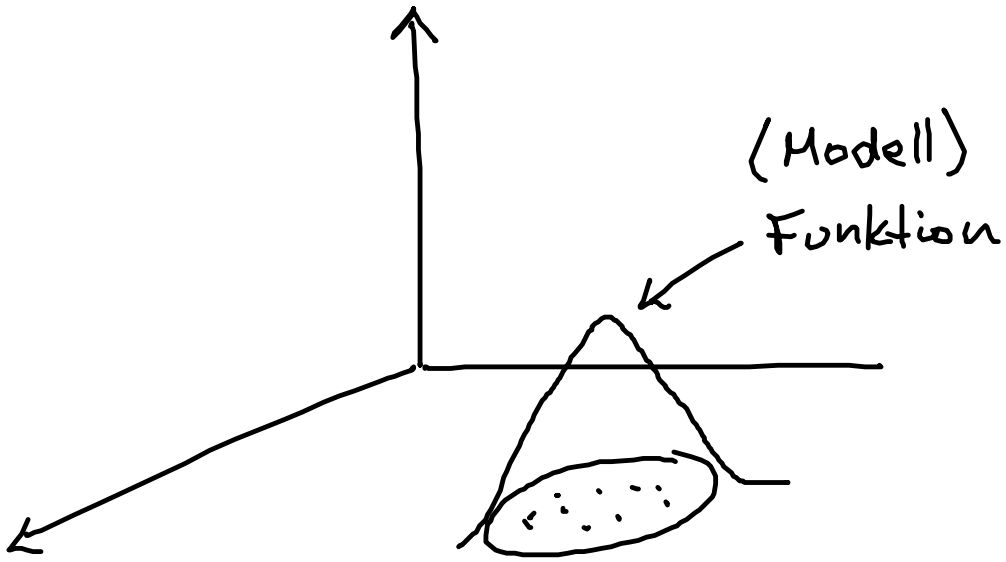
Wahrscheinlichkeit der Daten





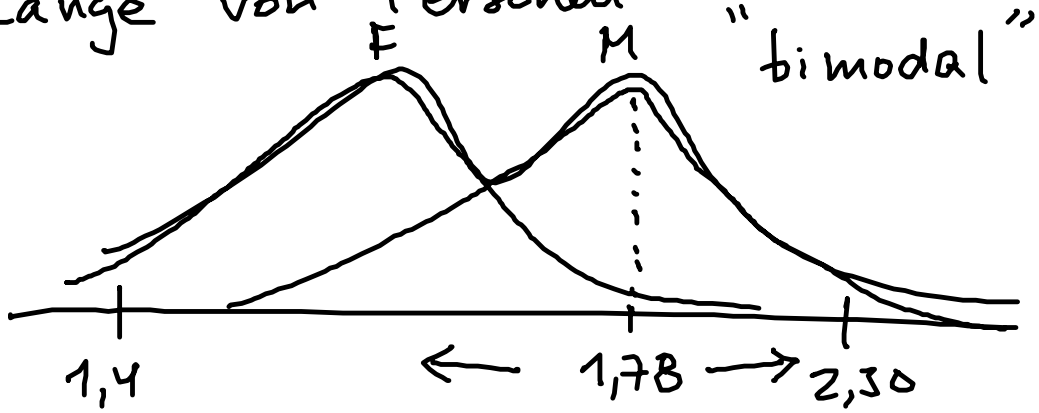
$10^5 \approx \frac{10^6}{10}$

Gebieten

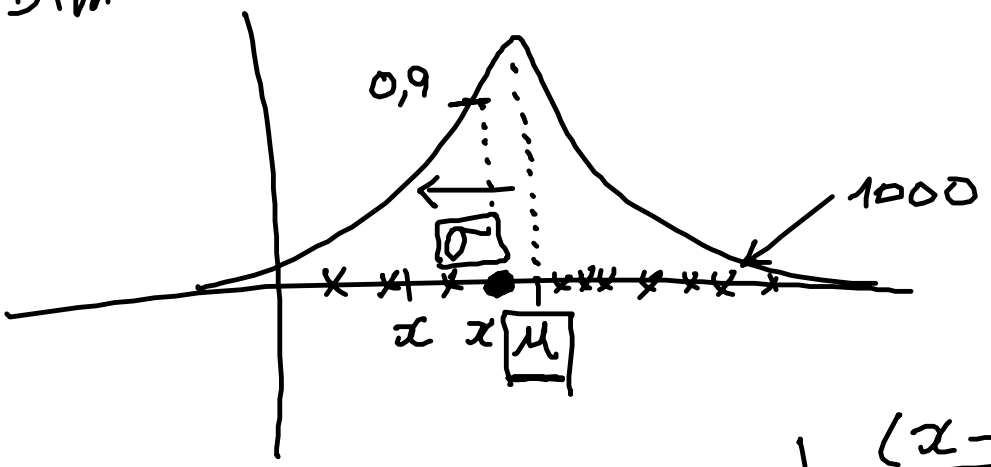


Normal Verteilung

Länge von Personen

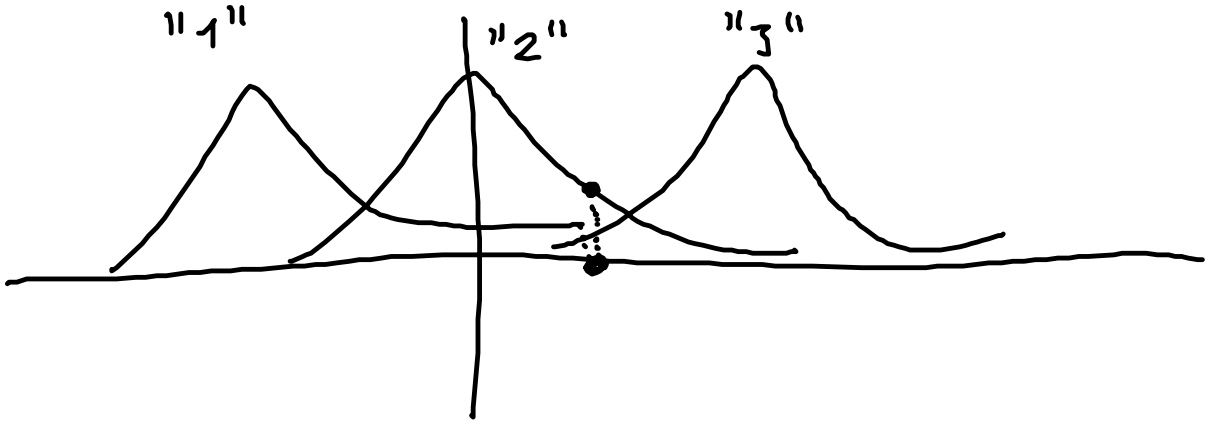


1-Dim



$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$p(x) = \ldots\ldots = 0,9$

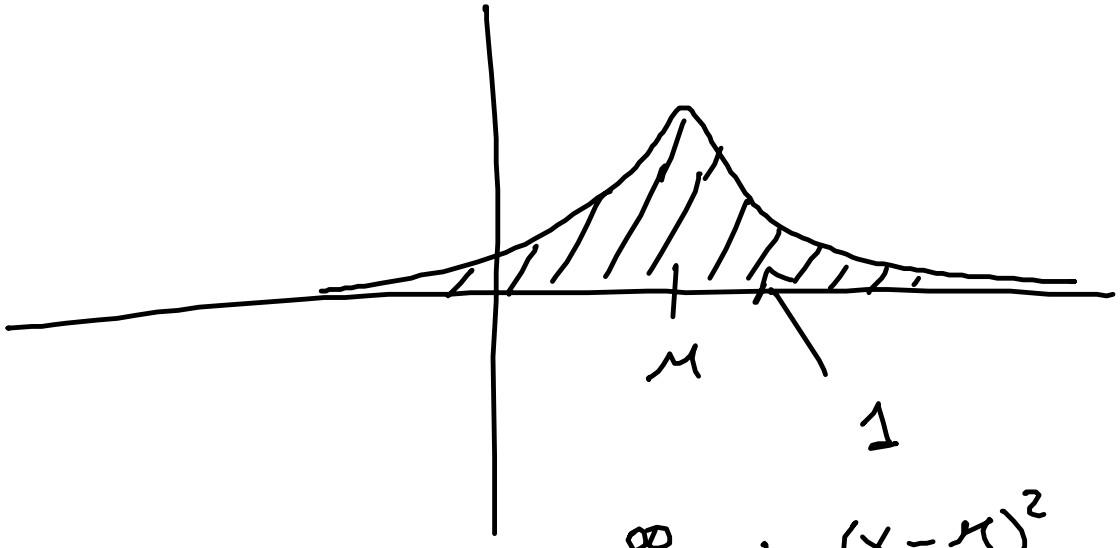


$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

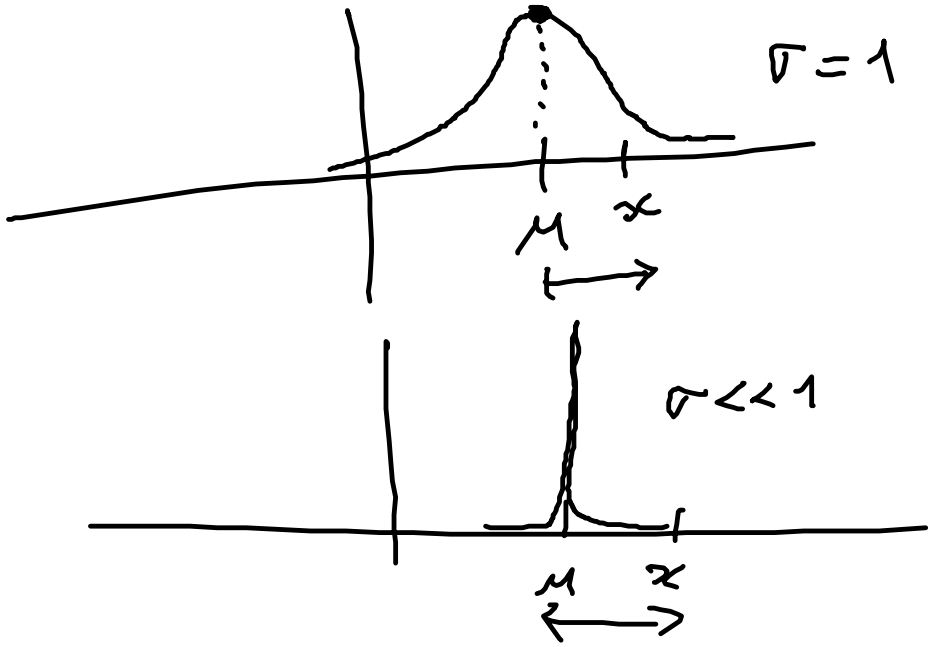
$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$

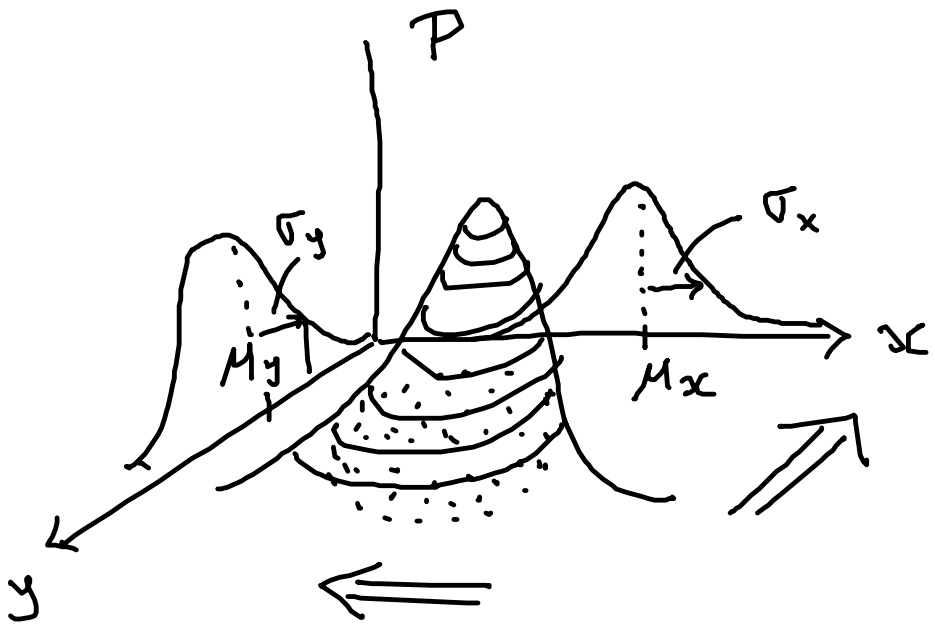


$$\sqrt{2\pi}\,\sigma = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



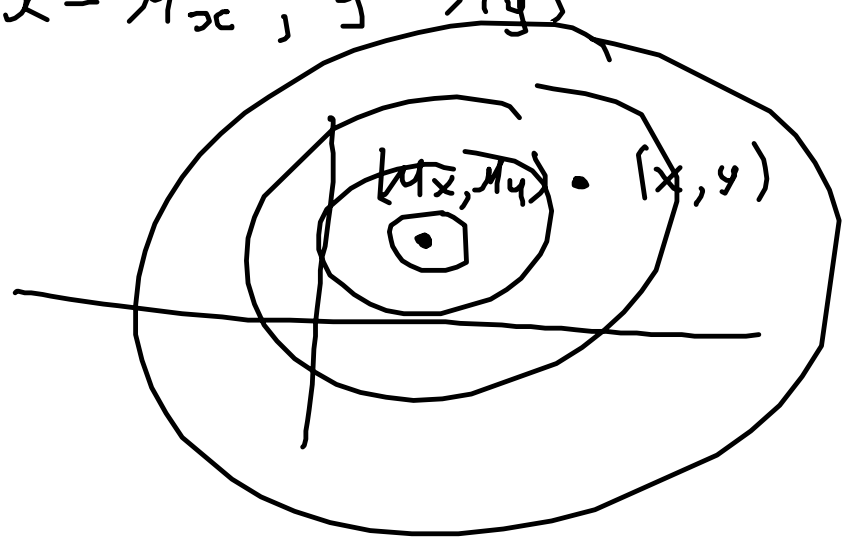
n-Dimensional



(x,y) Unbekannt

$$p(x) = k\, e^{-\text{Exponent}}$$

$$(x-\mu_x,\ y-\mu_y)$$



Exponent

$$-\frac{1}{2}(x-\mu_x,\ y-\mu_y)\begin{pmatrix}\frac{1}{\sigma_x^2} & 0\\ 0 & \frac{1}{\sigma_y^2}\end{pmatrix}\begin{pmatrix}x-\mu_x\\ y-\mu_y\end{pmatrix}$$

$$= -\frac{1}{2}\underbrace{\left(\frac{(x-\mu_x)^2}{\sigma_x^2}+\frac{(y-\mu_y)^2}{\sigma_y^2}\right)}$$

Skalar

$d = \text{Dim} = 2$

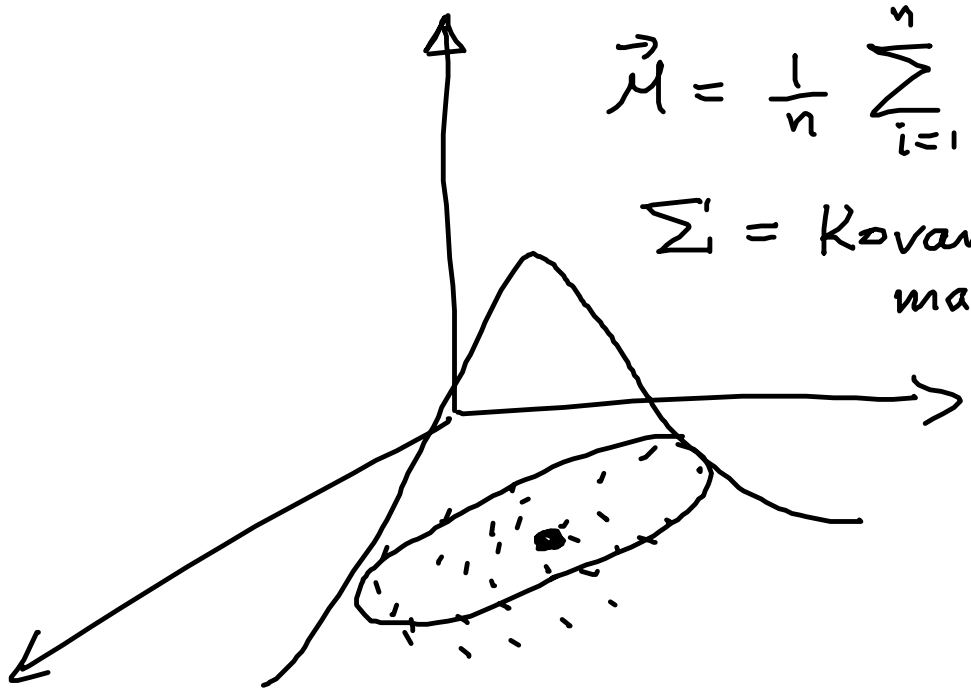
$$p(x,y) = \frac{1}{(2\pi)^{d/2} \|\Sigma\|^{1/2}} e^{-\frac{1}{2}(x-\mu_x, y-\mu_y)\Sigma^{-1}\begin{pmatrix} x-\mu_x \\ y-\mu_y \end{pmatrix}}$$

Multivariate Normalverteilung $d = 2$

$\vec{x} = (x_1, x_2, \dots, x_d)$ $\vec{\mu} = (\mu_1, \mu_2 \dots)$

$$p(\vec{x}) = \frac{1}{(2\pi)^{d/2} \|\Sigma\|^{1/2}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T \Sigma^{-1}}$$

$(\vec{x}-\vec{\mu})$



$$\vec{\mu} = \frac{1}{n} \sum_{i=1}^{n} \vec{x}_i$$

Σ = Kovarianz-matrix

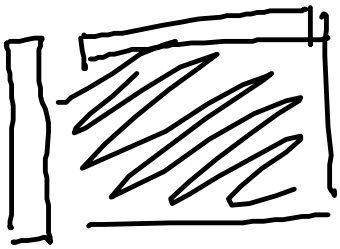
Zentriert

$\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$ Daten $\in \mathbb{R}^d$

Kovarianzmatrix

$$\Sigma = \frac{1}{n} \left(\vec{x}_1 \vec{x}_1^T + \vec{x}_2 \vec{x}_2^T + \dots \vec{x}_n \vec{x}_n^T \right)$$

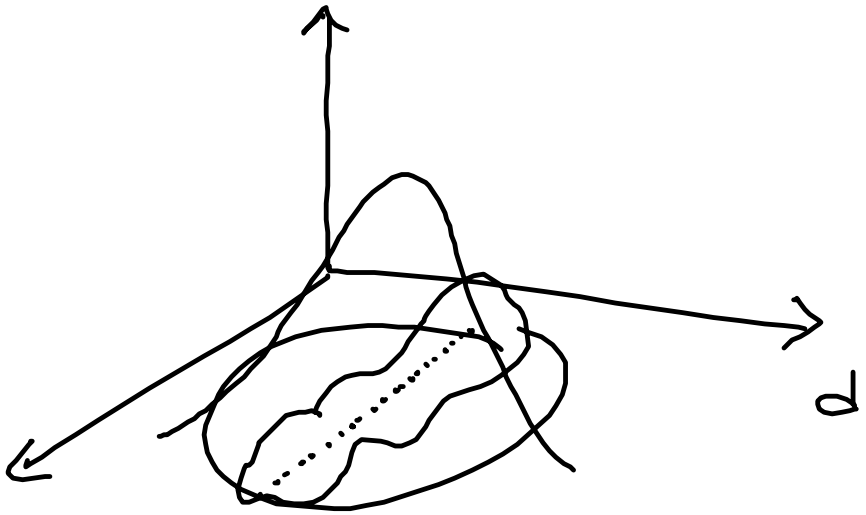
Dim d



$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1 \; 2 \; 3)$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Σ^{-1} = inverse von Σ



$$\Sigma = \frac{1}{n}\left((\vec{x}_1 - \vec{\mu})(\vec{x}_1 - \vec{\mu})^T + \dots \right)$$

2-Dim $\quad \vec{x}_1 = (x_{11}, x_{12})$

$$\Sigma = \frac{1}{n}\left(\begin{pmatrix} x_{11}-\mu_1 \\ x_{12}-\mu_2 \end{pmatrix} (x_{11}-\mu_1, x_{12}-\mu_2) \right.$$

$$\left. \dots \right)$$

$$= \frac{1}{n}\left\{ \begin{pmatrix} (x_{11}-\mu_1)^2 & (x_{11}-\mu_1)(x_{12}-\mu_2) \\ \text{''} & (x_{12}-\mu_2)^2 \end{pmatrix} \right.$$

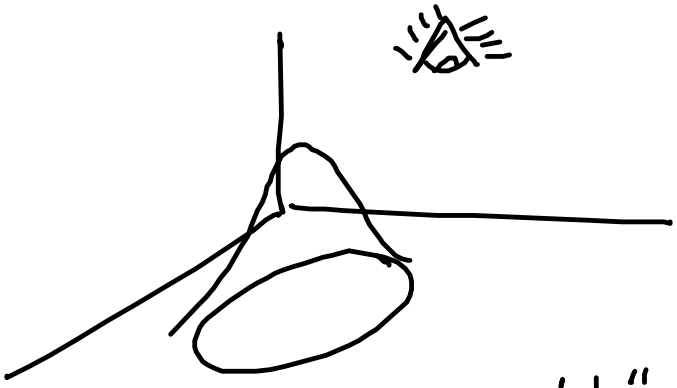
$$\left. + \dots \right\}$$

$$\Sigma = \frac{1}{n}\begin{pmatrix} \sum_{i=1}^{n}(x_{i1}-\mu_1)^2 & \text{crosstalk} \\ \text{crosstalk} & \sum_{i=1}^{n}(x_{i2}-\mu_2)^2 \end{pmatrix}$$

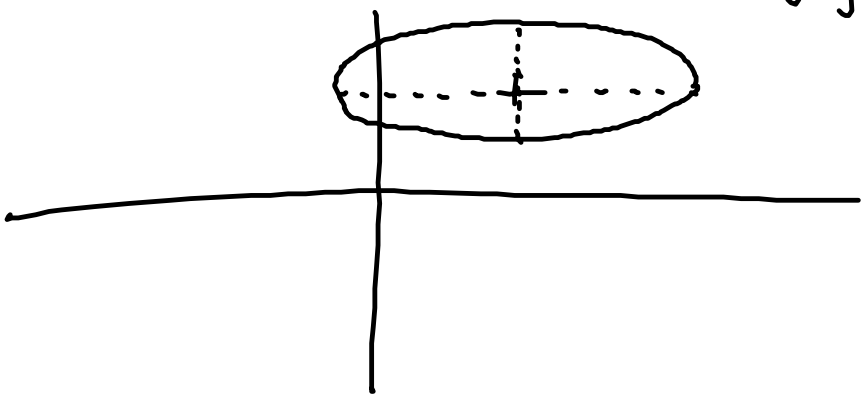
Für unabhängige Komponenten

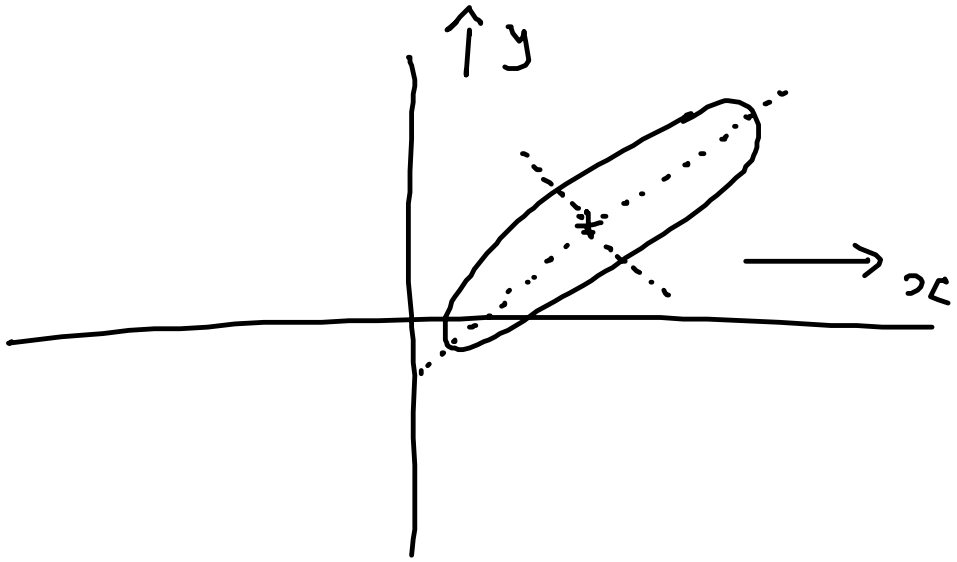
$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$$

$$\Sigma^{-1} = \begin{pmatrix} 1/\sigma_1^2 & 0 \\ 0 & 1/\sigma_2^2 \end{pmatrix}$$

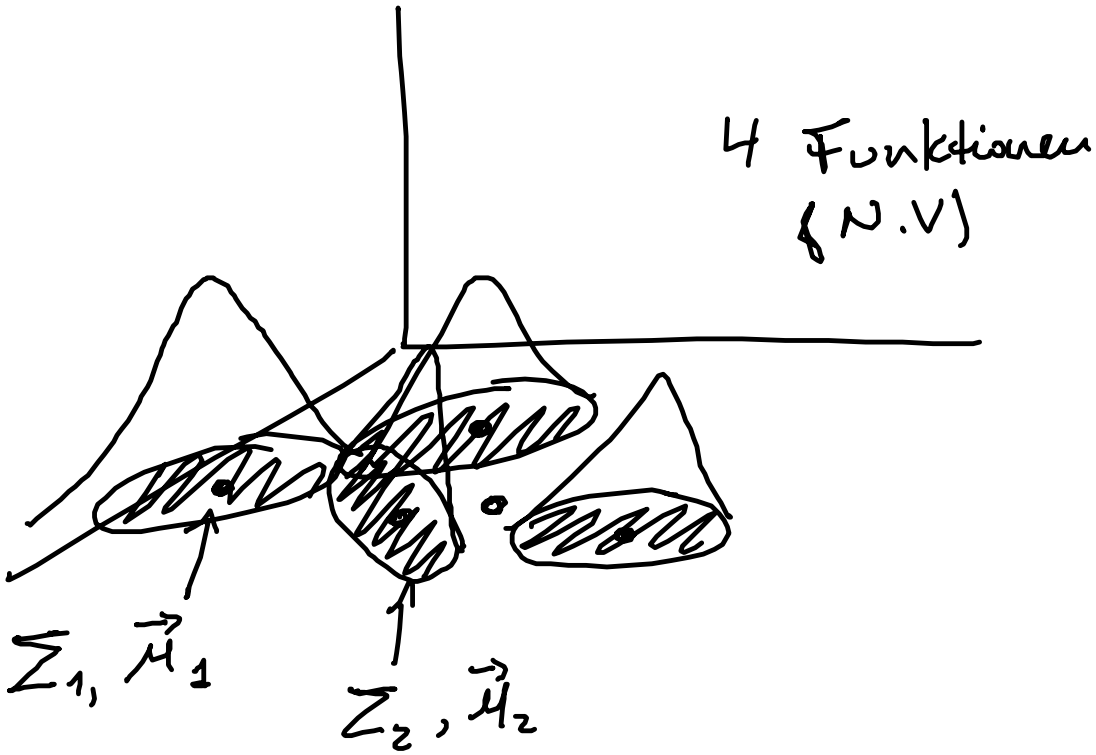


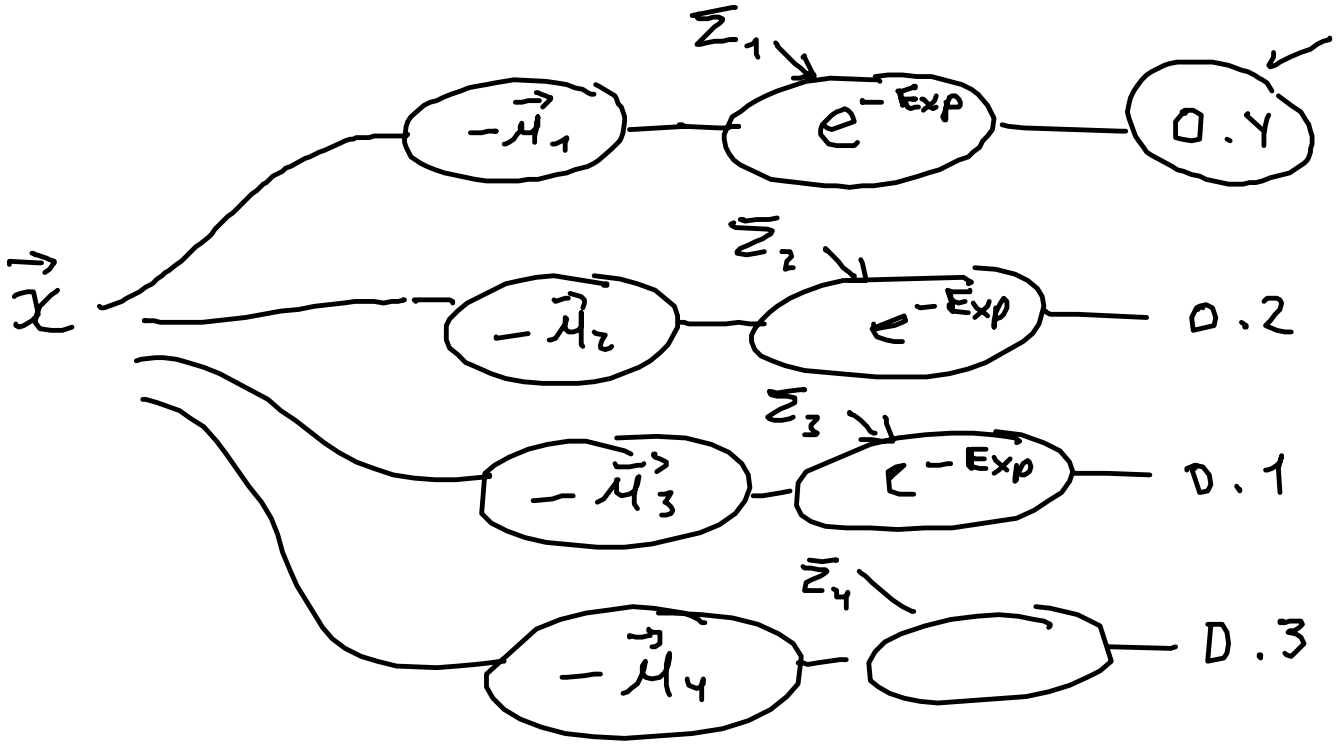
unabhängig





Klassifikator mit Gauss-Kurven

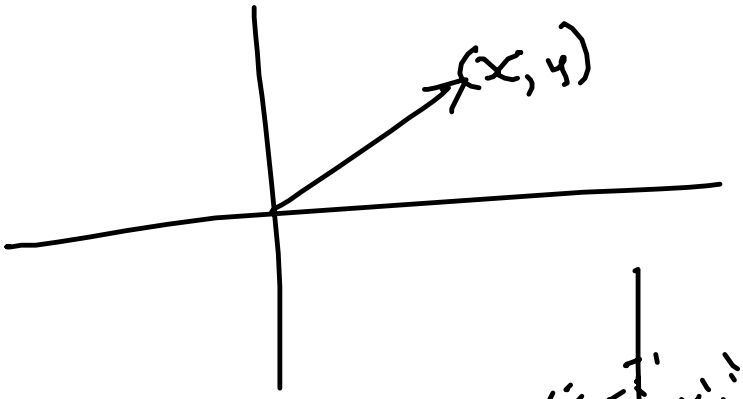




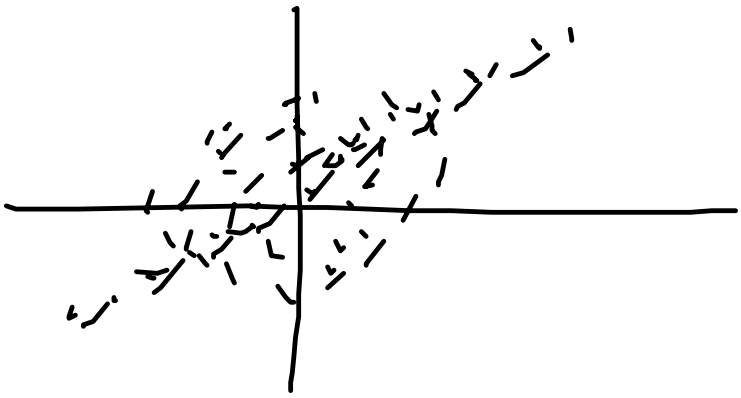
Radial-Basis-Functions

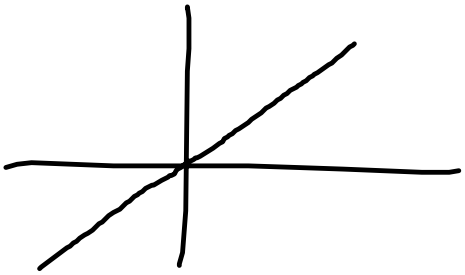
RBF-Klassifikator

—∘—

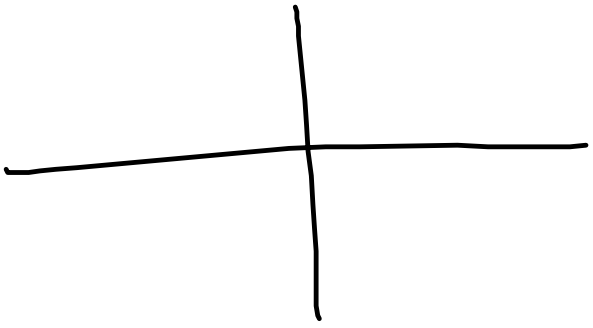


(x, y)





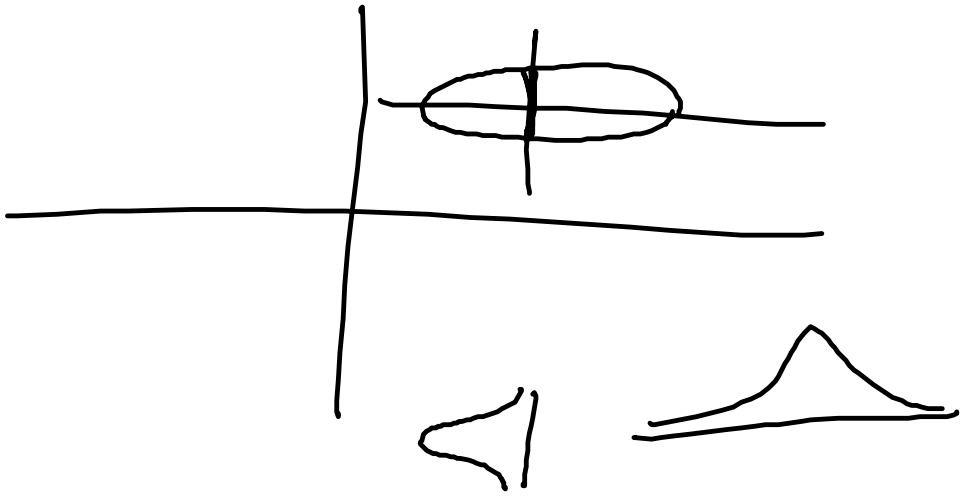
$$(x-\mu)^T \sum^{-1} (x-\mu) \qquad \vec{\mu} = 0$$

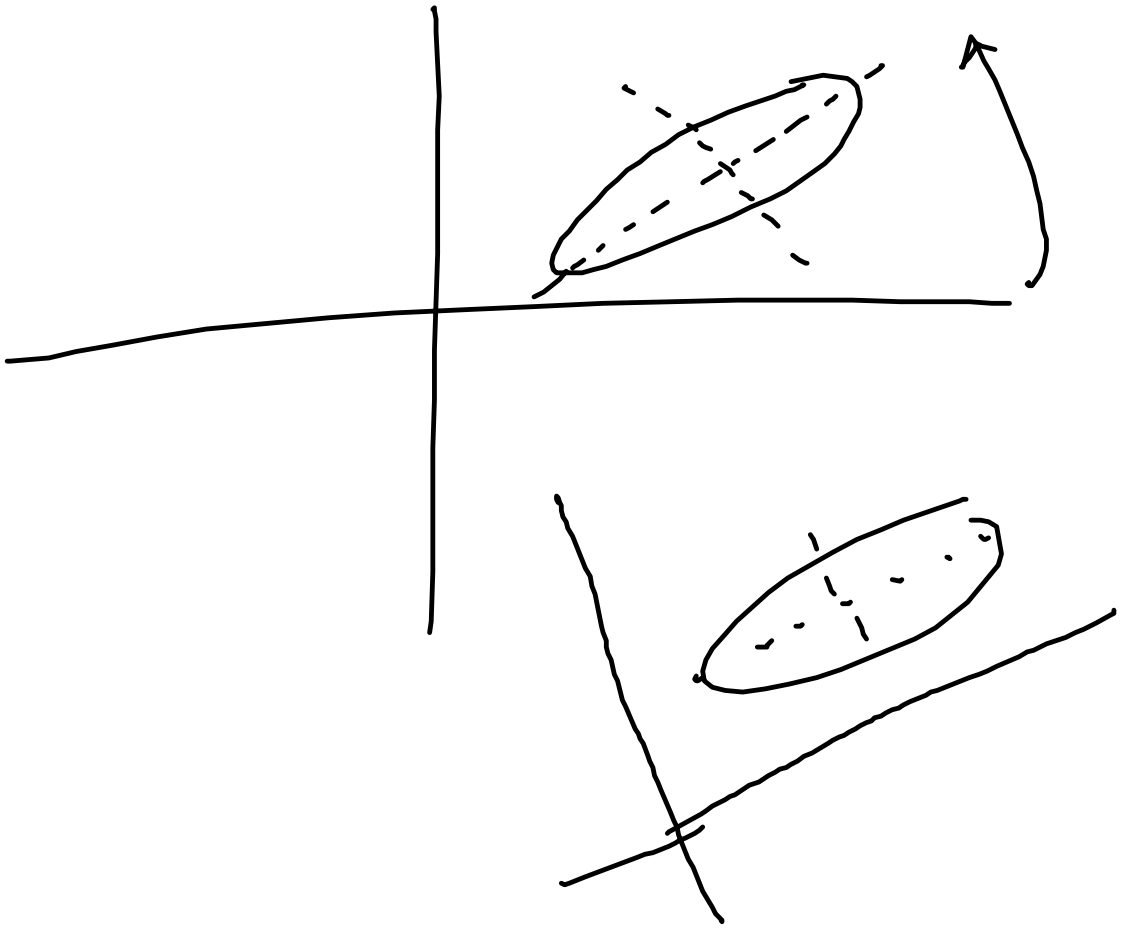


$$\vec{x}^T \sum^{-1} \vec{x}$$

Q.F.

$\vec{x}^T \Sigma \vec{x}$





$$\underbrace{\vec{x}^T \Sigma^{-1} x}$$

$$\vec{x}^T \underbrace{R^T D R} \vec{x}$$

$$\Sigma^{-1} = \begin{pmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \frac{1}{\sigma_2^2} & \\ & & \ddots \\ 0 & & \frac{1}{\sigma_n^2} \end{pmatrix}$$