Models for DC Motors

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Abstract. This document describes how to model a DC motor, the acceleration curve for the angular velocity, the acceleration due to PWM signals, and how our robot wheels accelerate.

1 Introduction

It is important to know how DC motors can accelerate a robot. The final velocity of a DC motor depends on the voltage applied to it, the motor's characteristics, and the load on the wheels. Also,sometimes the wheels do not have enough grip and when the motor rotates faster, the wheels begin slipping. Knowing the form of the acceleration curve of a wheel rotating without slipping is useful for implementing controllers which provide traction control.

We consider DC motors with permanent magnets. The force provided by the motors is a torque. The stall torque listed in DC motors data sheets is the maximum torque produced by the motor when the rotator is not held and is prevented from rotating.

The rotator in DC motors rotates because the current flowing through a coil produces a magnetic field repelled by permanent magnets mounted in the armature. When the coil rotates brushes touching the coil contacts invert the polarity of the voltage in such a way that the coil is repelled again, and so on.

When the motor is rotating, the voltage V applied to the motor is diminished by the voltage E produced by the rotator (which behaves like a dynamo when rotating). The faster the rotator rotates, the larger the value of E. In general

$$E = k_e \omega$$

where k is a constant, and ω is the angular velocity of the motor. The generated voltage E increases proportionally to the angular velocity. When E comes close to V, the motor is rotating fast but the current flowing through the coil is lower. This means that the force acting on the rotator is also lower. When eventually $V - E = 0$, the current flowing through the coil drops to zero and there is no torque acting on the motor. The DC motor has reached its final velocity.

The torque on the rotator (the useful torque for a machine) is proportional to the current in the coil:

$$T = k_t I$$

where k_e is a constant. In SI units, $k_e = k_t$. Since $I = (V - E)/R$, where R is the resistance of the coil, we obtain

$$T = \frac{k_t}{R}(V - E) = \frac{k_t}{R}(V - k_e\omega)$$

Therefore, there is an inverse linear relationship between the torque of the motor and its angular velocity. Fig. 1 shows a graph of torque versus RPM for a certain motor.

The free RPM is the number of revolutions per minute that a free running DC motor can achieve. The free RPM is determined by the autogenerated voltage and the friction in the motor.

2 A correction: autoinductivity

Actually, a correction has to be made to the last expression. The voltage generated by the motor coil is produced because the coil is going through the magnetic flux lines of the permanent magnet. If the current in the coil is changing, then the magnetic field generated by the coil is also changing. The magnetic flux of the field generated by the own current is varying, and this is equivalent to a movement of the coil through a magnetic field. This means that variations in the current through the coil lead to a generated voltage V_L given by

$$V_L = L\frac{dI}{dt}$$

where L is the autoinductivity constant of the coil, and dI/dt the derivative of the current with respect to time. The autoinduced voltage opposes the voltage V applied to the motor, and therefore it can be added to E. The last equation transforms into

$$T = \frac{k_t}{R}(V - E - L\frac{dI}{dt}) = \frac{k_t}{R}(V - k_e\omega - L\frac{dI}{dt})$$

3 Cases at the boundary

When ω is zero, and the current is stable, we have the stall torque T_s which is then

$$T_s = \frac{k_t V}{R}$$

When the torque is zero, and the current stable, the motor has reached its final angular velocity

$$\omega_f = \frac{k_t V}{k_e R}$$

We can write k_e as

$$k_e = \frac{k_t V}{\omega_f R} = \frac{T_s}{w_f}$$

and we can rewrite the original expression for T as

$$T = T_s(1 - \frac{\omega}{\omega_f})$$

under the assumption that dI/dt is zero. As the equation shows, the torque decreases proportionally to the factor $\frac{\omega}{\omega_f}$.

4 The motion equation

Let us call J the moment of inertia of the wheel and the motor rotator. Newtonian mechanics let us write a differential equation for the wheel's rotation. We will set autoinductivity to zero, for simplicity.

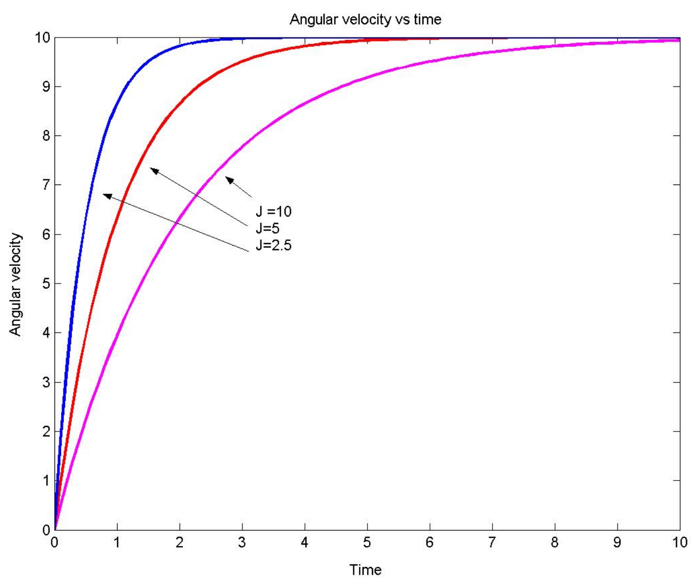

Fig. 1. Convergence to the final angular velocity under different loads

The equation of motion for the wheel $J\dot{\omega} = T$, can be written as

$$J\dot{\omega} = T_s(1 - \frac{\omega}{\omega_f})$$

or simply

$$\dot{\omega} = \frac{T_s}{J}(1 - \frac{\omega}{\omega_f})$$

The solution to this differential equation is

$$\omega = \omega_f (1 - e^{-\frac{T_s}{J\omega_f} t})$$

From this expression we see that the angular velocity increases nonlinearly, along an exponential function, and reaches asymptotically the maximum velocity ω_f.

Fig. 1 shows how the angular velocity converges to its final value asymptotically. When the load diminishes (the moment of inertia J is smaller) convergence is faster.

5 PWM signals

A motor can be controlled by reducing the voltage applied to the rotor. This reduces the stall torque and the maximum achievable velocity. However, there are two problems with this approach. The first problem is that a digital command from the microprocessor has to be transformed into an analog value (the voltage), and the second is that at low voltages the stall torque could not be enough to start a wheel moving. Friction is larger when the rotor is not moving as when it is moving.

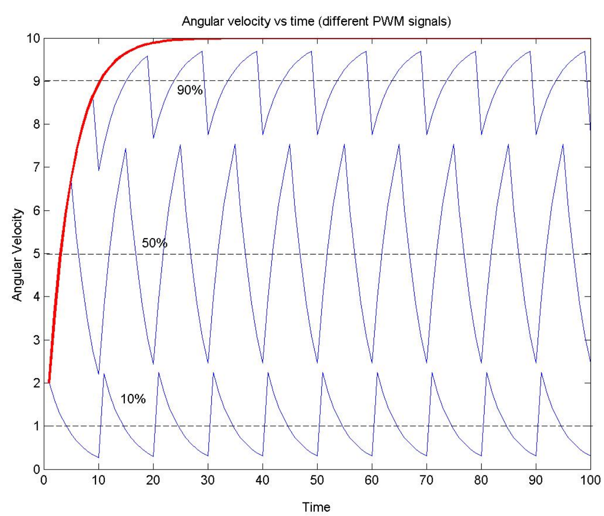

Fig. 2. Acceleration of a DC motor under different PWM signals. The broken lines show the mean velocity produced by each PWM signal.

To solve this problem, pulse width modulation is used. Instead of applying to the motor a 50% reduced voltage, the maximum voltage is applied but only half of the time. A carrier frequency is selected, and at only 50% of the cycles

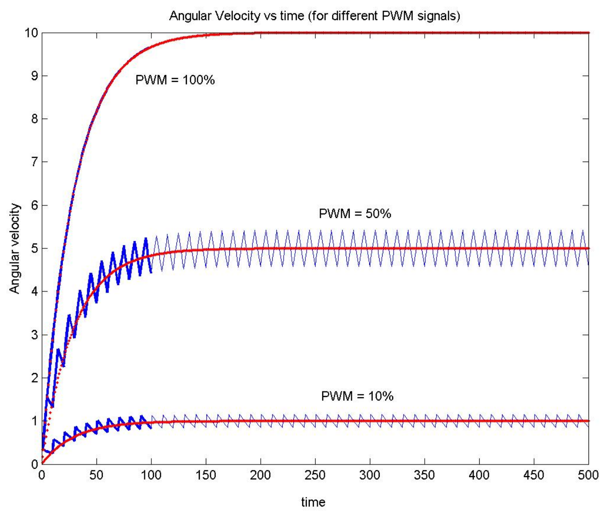

Fig. 3. Acceleration of a DC motor under different PWM signals.The solid lines show the mean velocity produced by each PWM signal.

the maximum voltage is applied. If only 10% of the cycles are used to apply a voltage, the PWM signal is said to be set at 10

Fig. 2 shows what happens when a PWM signal with 100%, 90%, 50%, and 10% duty cycle is applied. The motor is accelerated in discrete steps. When the maximum voltage is applied, the velocity of the motor increases along the exponential curve derived above. When the voltage is cut-off, the motor decelerates due to the self-generated voltage E. The total effect is series of acceleration and deceleration steps, which produce a similar effect to a reduction of voltage to the applied PWM level. However, the maximum possible torque for each velocity is always applied. Motors can rotate at low velocities and can be controlled better.

Fig. 2 exaggerates the control peaks. Usually the frequency used for the PWM signal is high, in the ultrasonic region, so that people cannot hear it. When the PWM signal is applied with a low frequency it is possible to hear the a characteristic buzzing sound.

Fig. 3 shows the PWM signal during a longer period of time. The inertia of the wheel and damping factors all contribute to smooth the real movement of the wheel, which theoretically proceeds in discrete jumps.

6 Our omnidirectional wheels

Our omnidirectional robots use four omnidirectional wheels with axis at an angle of 60 degrees (not 90) with respect to the front-to-rear symmetry axis of the robot (Fig. 4 shows one of our robots with three wheels) . The wheels contain small passive wheels, 12 in one of our designs, and up to 30 in another. Fig. 5

shows a picture of our wheels with 12 small passive wheels. Fig. 6 shows the omnidirectional wheel with 30 passive wheels.

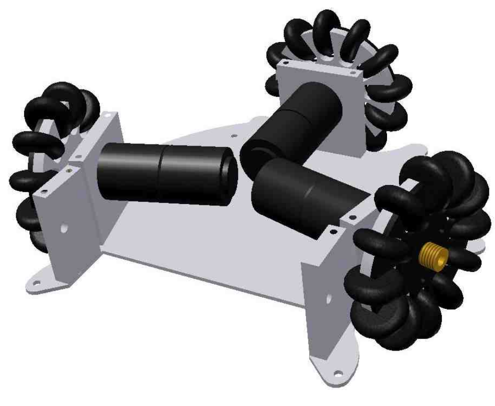

Fig. 4. The chassis of a robot with three omnidirectional wheels.

The purpose of the passive wheels is to allow the robot to slide without friction in the direction orthogonal to a wheel. When three or more of these wheels are combined in a robot, the robot can drive in straight line in any direction without rotating. It can also drive in a straight line and rotate along the line, in order to arrive at any desired angle at its final position. Omnidirectional wheels have been used for many years in industrial vehicles. They allow careful positioning and driving.

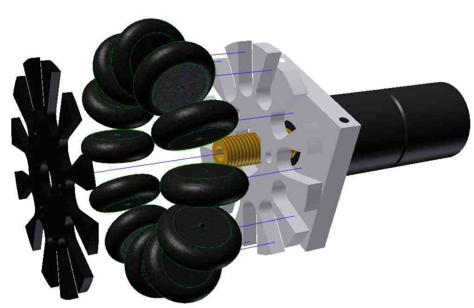

Fig. 5. Explosion diagram of one of our omnidirectional wheels with 12 passive wheels.

Fig. 6. Our omnidirectional wheel with 30 passive wheels.

The wheel with the smaller subwheels has better contact with the floor. Usually, two or three wheels are in contact with the carpet and provide traction for the wheel. The wheels with 12 passive subwheels provide less traction. They can be modelled as a dodecagon (a polygon with 12 sides). When the wheel rotates, it pushes the robot sligthly up and down and energy is lost. Also, when traction changes from one of the passive wheels to the next, the wheel has less traction and rotates with minimal contact with the floor.

The problem of having wheels which are not exactly circular can be seen in the next diagrams of wheel speed for a robot with four omnidirectional wheels, two in front, two in the back of the robot.

When the robot accelerates forward, the two rear wheels carry most of the load. The front wheels are lifted and lose traction. This means that the wheels accelerate free, and when the front of the robot hits the ground again, they decelerate. Unfortunately, this happens several times, until the front wheels get full traction.

Fig. 7 shows four curves for the angular velocity of four wheels. Two wheels have negative velocity, because the motor rotates in the negative direction. The curve shows that the two front wheels lose traction, specially at the beginning of the movement, and accelerate like wheels without load. When the wheel ragains traction (because the next passive wheel touches the ground), the front wheels decelerate to the velocity of the rear wheels. Then the process is repeated. When the robot is moving at a certain speed, all four wheels have good traction.

The x axis units are control cycles of 8 ms each. The y axis is number of pulses sent by the pulse counters in each wheel. No wheel sends more than 100 pulses per control cycle. The curve shows that traction with the four wheels is achieved after around 70 control cycles, that is, 568 ms. The robot achieves its final velocity after around 160 cycles, that is, 1.28 seconds.

Losing traction on the wheels is not a big problem, when the lose of traction is symmetrical, that is, when it does not affect the direction in which the robot starts moving. Usually though, the wheels' traction is different and the robot

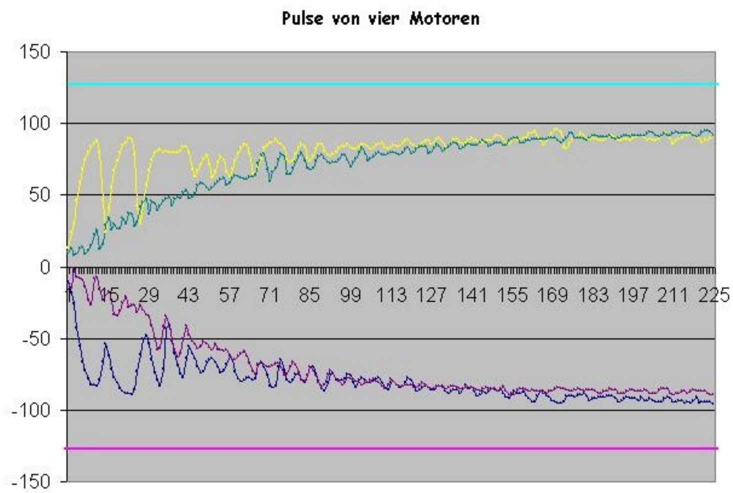

Fig. 7. The pulses sent by four motors in an omnidirectional robot with four wheels. One front wheel and one rear wheel have positive tick counts, one front and one rear wheel negative tick counts.

starts moving at an angle different to the intended one. The computer vision and control loop must then correct the problem.

In Formula 1 cars, the traction of the wheels is monitored by special electronics. Whenever a wheel loses traction, power is cut to the motor. This could be also done with our robots, using the microcontroller.

7 Project

Program the microcontroller so that traction control is implemented and the wheels do not slip on the floor when starting the robot.