13. Problem Set for

Advanced Algorithms II — Spring 2017

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Due 17. July 2017, in class.
This is the last problem set.

Problem 1 Chromatic number — *10 points*

Let $G = (V, E)$ be a graph, and let $c \in \mathbb{N}$. A *c-coloring* of G is a function $\chi : V \to \{1, \dots, c\}$ such that $\chi(u) \neq \chi(v)$ for all edges uv of G. The *chromatic number* of G, $\chi(G)$, is the smallest $c \in \mathbb{N}$ such that there is a c-coloring of G.

(a) Show that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that $\chi(G) \leq f(\text{tw}(G))$.

(b) Suppose we are given a non-redundant tree-decomposition $T = (W, F)$ and $(V_w)_{w \in W}$ for G of width $\text{tw}(G)$. Show that there is an algorithm to compute a coloring of G with $\chi(G)$ colors that runs in time $g(\text{tw}(G)) \operatorname{poly}(|V|)$, where $g : \mathbb{N} \to \mathbb{N}$ is a fixed function.

Problem 2 Tree-Width of the triangulated regular n-gon — *10 points*

Let $n \geq 3$. The *regular n-gon* Q_n is the plane graph with vertices $\{v_0, \dots, v_{n-1}\}$ and edges $\{v_i v_{i+1} \mid i = 0, \dots, n-2\} \cup \{v_{n-1} v_0\}$ such that $v_i = (\cos(i2\pi/n), \sin(i2\pi/n))$, for $i = 0, \dots, n-1$. A *triangulation* R_n of Q_n is obtained from Q_n by adding a maximal set of edges $v_i v_j$, $0 \leq i < j \leq n-1$, to Q_n without creating any crossings. It is known that any triangulation R_n of Q_n has exactly $2n-3$ edges and $n-1$ faces, and that all inner faces of R_n are triangles.

Let R_n be a triangulation of Q_n. Show that $\text{tw}(R_n) = 2$, and give a polynomial-time algorithm to obtain a non-redundant tree decomposition of R_n with width 2.

Problem 3 Separated sets — *10 points*

Let $G = (V, E)$ be a graph with n nodes and m edges. Let $Y, Z \subseteq V$ with $|Y| = |Z| = k$. Give an algorithm that determines in $O(km)$ time whether there is a set $S \subseteq V$ with $|S| < k$ such that $G \setminus S$ contains no path between $Y \setminus S$ and $Z \setminus S$.