12. Problem Set for

Advanced Algorithms II — Spring 2017

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Due 10. July 2017, in class.
This is the penultimate problem set.

Problem 1 Tree-width of simple graphs *10 points*

(a) Let $G = (V, E)$ be a tree with at least two nodes. Show that $\text{tw}(G) = 1$.

(b) Let $G = (V, E)$ be a cycle with $n \geq 3$ nodes. Show that $\text{tw}(G) = 2$.

Problem 2 Properties of tree-width *10 points*

(a) Let $G = (V, E)$ be a connected graph, and let H be a subgraph of G (not necessarily induced). Show that $\text{tw}(H) \leq \text{tw}(G)$.

(b) Let $G = (V, E)$ be a connected graph with $\text{tw}(G) = 1$. Show that G is a tree.
Hint: Use Problem 1(b) and (a).

Problem 3 Separators from tree-decompositions *10 points*

Let $G = (V, E)$ be a connected graph, and let $T = (W, F)$ and $(V_w)_{w \in W}$ be a tree-decomposition for G. Let $f = xy$ be an edge of T, and let T_x, T_y be the components of $T \setminus f$. Set $X = \bigcup_{w \in T_x} V_w$ and $Y = \bigcup_{w \in T_y} V_w$, and let $G_x = G_{|X}$ and $G_y = G_{|Y}$ be the corresponding induced subgraphs of G.

Show that $G_x \setminus (V_x \cap V_y)$ and $G_y \setminus (V_x \cap V_y)$ have no common vertices and that there is no edge in E between them.