6. Problem Set for

Advanced Algorithms II — Spring 2017

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Due 29. May 2017, in class

Problem 1 KKT-Algorithm *10 Points*

In this problem, we will see an alternative proof of the KKT-sampling lemma. We consider the variant of the algorithm where the sample $R \subseteq E$ is obtained through *Bernoulli-sampling*: each edge $e \in E_1$ is added to R independently with probability $1/2$.

(a) Argue that the following algorithm gives the same distribution on edge sets F_2 as the algorithm that first chooses R and then computes $\text{MSF}(V_1, R)$.

Set $A = \emptyset$. Sort the edges in E_1 according to their weight. For each edge $e \in E_1$, from lightest to heaviest, do the following: flip a fair coin. If the coin flip yields tails, then discard e. If it yields heads, then add e to A if and only if it does not create a cycle in (V_1, A). Finally, set $F_2 = A$.

(b) Let $e_i \in E_1$ be the ith edge in the sorted order of edges from E_1. Argue that after the $(i-1)$th coin flip, it is determined whether e_i will be F_2-light or not.

(c) Divide the random experiment from (a) into *phases*. The kth phase begins as soon as the kth edge is inserted in to A. Let X_k be the number of F_2-light edges that are created during phase k. Give an upper bound for the expected value of X_k.

(d) Show that the expected number of F_2-light edges is at most $2|V_1|$.

Problem 2 MST for planar graphs *10 Points*

Let $G = (V, E)$ be a simple connected planar planar graph with n vertices. Suppose that the edges of G are weighted and that all weights are pairwise distinct. Show that $\text{MST}(G)$ can be computed in $O(n)$ time.

Hint: Show that if we contract an edge in a planar graph, the resulting graph is again planar.

Problem 3 Cuckoo-Hashing *10 Points*

Prove the theorem from class that a set $S \subset U$ can be inserted successfully for the hash functions $h_1, h_2 : S \to \{0, \ldots, m-1\}$ if and only if the corresponding cuckoo graph $G_{h_1,h_2}(S)$ is valid.