4. Problem Set for

Advanced Algorithms II — Spring 2017

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Due 15. May 2017, in class

Problem 1 Farkas' Lemma: Base Case *10 Points*

Let $A \in \mathbb{R}^{m\times 1}$ and $\mathbf{b} \in \mathbb{R}^m$. Suppose that there is no $x \in \mathbb{R}$ with $x \geq 0$ and $Ax \leq \mathbf{b}$. Show that then there is a $\mathbf{y} \in \mathbb{R}^m$ with $\mathbf{y} \geq \mathbf{0}$, $\mathbf{y}^T A \geq 0$ and $\mathbf{y}^T\mathbf{b} < 0$.

Problem 2 Fourier-Motzkin Elimination I *10 Points*

Consider the following system of linear inequalities with five constraints and three variables.

$$\begin{aligned} 2x - 5y + 4z &\leq 10 \\ 3x - 6y + 3z &\leq 9 \\ 5x - 10y - z &\leq 15 \\ -x + 5y - 2z &\leq -7 \\ -3x + 2y + 6z &\leq 12 \\ x, y, z &\geq 0 \end{aligned}$$

Use Fourier-Motzkin elimination to remove x. Is the system feasible?

Problem 3 Fourier-Motzkin Elimination II *10 Points*

Describe an algorithm for solving LPs that is based on Fourier-Motzkin elimination. Give a good upper bound on the running time of your algorithm.

Hint: The lemma from class deals only with the feasibility of LPs. How can you get from the decision version to the optimization?

Problem 4 Proof of Farkas' Lemma *optional, 10 Extra Points*

Prove that the two alternatives in Farkas' Lemma cannot hold simultaneously.