2. Problem Set for

Advanced Algorithms II — Spring 2017

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Due 02. May 2017, 12:00, in the mailbox next to room 111

Problem 1 Quadratic Programming *10 Points*

Suppose that in the constraints of a linear program, we also allow summands of the form $d_{ij}x_j^2$, for $d_{ij} \in \mathbb{R}$. Show that the resulting algorithmic problem is NP-hard.

Problem 2 Variants of LPs *10 Points*

In class, we defined a linear program as a constrained optimization problem of the following form

$$\max c^T x, \text{ subject to}$$
$$Ax \leq b$$
$$x \geq 0.$$

Show that the following variants and generalizations can all be reduced to an LP of this form:

(a) the objective function is of the form $\min c^T x$,

(b) there is a constraint of the form $a_i x \geq b$,

(c) there is a constraint of the form $a_i x = b$,

(d) the variable x_i may be positive or negative,

(e) there is a constraint of the form $x_i \geq |x_j|$.

Problem 3 LP-Duality *10 Points*

Let $G = (V, E)$ be a bipartite graph. A *matching* is a set $M \subseteq E$ of edges such that each vertex in G is incident to at most one edge in M. A *maximum matching* is a matching of maximum cardinality.

(a) Draw a bipartite graph with 10 vertices that has a maximum matching with 5 edges. Your graph should also contain a *maximal* matching with 4 edges. (A matching M is *maximal* if there is no matching M' with, $M' \supset M$ and $M' \neq M$.)

(b) Give an LP-formulation for the problem of finding a maximum matching in a bipartite graph G. Briefly explain your LP in words.

(c) Give the dual LP for your LP from (b). Interpret the result.

(d) Read up on *König's theorem*. Is the theorem already implied by your results from (b) and (c)? Explain.