6. Assignment on

Computational Complexity Theory

SoSe 2016

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Due on 7. June 2016 in the tutorial session

Problem 1 PSPACE and PH *10 points*

(a) Complete our proof that TQBF is PSPACE-complete by showing that we can convert the resulting formula to prenex form in polynomial time.

(b) Show that if PH has complete problems, then the polynomial hierarchy collapses. What does this say about the conjecture PSPACE = PH?

Problem 2 Certificate Definition of NL *10 points*

Consider the following attempt to define NL using certificates:

A language $L \subseteq \{0,1\}^*$ is in bogusNL if and only if there exists a deterministic *logspace* Turing machine V and a constant $c \geq 1$, such that for every $w \in \{0,1\}^*$, we have

$$w \in L \quad \Leftrightarrow \quad \exists y \in \{0,1\}^*, |y| \leq |w|^c : V(w,y) \text{ accepts.}$$

(a) Show that SAT $\in$ bogusNL. Is it likely that bogusNL = NL?

(b) Explain what is wrong with the definition of bogusNL.

(c) Give a better definition of NL in terms of certificates. Prove that your definition and the definition you saw in class are equivalent.
Hint: Use a special tape for the certificate.

Problem 3 Alternation *10 points*

An *alternating* Turing machine (ATM) $M = (Q, \Gamma, \delta_0, \delta_1)$ is defined as follows: as usual, M is a multitape Turing machine with input alphabet $\{0,1\}$ and tape alphabet Γ. Every state in Q is either *existential* or *universal*. There are three distinct special states $q_{\text{start}}, q_{\text{yes}}, q_{\text{no}} \in Q$. In each step, M can choose between δ_0 or δ_1 for the next transition. The ATM accepts a word w if whenever M is in an existential state, there *exists* a choice of transition function that eventually leads to q_{yes}, and if whenever M is in a universal state *both* choices of transition function eventually lead to q_{yes}.

(a) Interpret ATMs in terms of a binary tree that represents the choices of transition function.

(b) The *alternating time* $\mathrm{atime}_M(w)$ for an ATM M on input w is the *maximum* number of steps over all transition function choices until M reaches a halting state q_{yes} or q_{no}. As usual, we define $\mathrm{atime}_M(n)$ as the maximum of $\mathrm{atime}_M(w)$ over all $w \in \{0,1\}^n$.

Give easy interpretations of NP and coNP in terms of alternating TMs.

(c) We define AP as the class of all languages that can be decided by an ATM in polynomial time. Show that AP $=$ PSPACE.

Hint. For AP $\subseteq$ PSPACE, show that every polynomial-time alternating Turing machine can be simulated with polynomial space. For PSPACE $\subseteq$ AP, begin by showing that there is an ATM that decides TQBF in polynomial time.