Zettel 4

MafI 2, SoSe 2013, Tutorium [name unclear] (Musterlösung)

graphisch: $-1+i$, $\varphi = \frac{\pi}{2}+\frac{\pi}{4} = \frac{3\pi}{4}$, $r=\sqrt{2}$

A1. a) $(-1+i)^{10} \overset{\circ}{=} \left(\sqrt{2}\, e^{i\frac{3\pi}{4}}\right)^{10} = 2^5 e^{i\pi\frac{3}{4}\cdot 10} = 32 e^{i\pi\frac{15}{2}} = 32 e^{i\pi\left(6+\frac{3}{2}\right)} = 32 \underbrace{e^{i\pi 6}}_{=1} \cdot e^{i\pi\frac{3}{2}} = 32 \underbrace{e^{i\pi\frac{3}{2}}}_{=-i} = \underline{\underline{-32i}}$

oder: $(-1+i)^{10} = \left((-1+i)^2\right)^5 = (1-2i-1)^5 = (-2i)^5 = -2^5 i^5 = -2^5 i = \underline{\underline{-32i}} = \ldots$

b) $(1-i\sqrt{3})^7$. Bestimme Polardarst. von $1-i\sqrt{3}$.

$$r = \sqrt{1+\sqrt{3}^2} = \sqrt{4} = 2$$

$$\varphi = -\arccos\left(\frac{1}{2}\right) = -\frac{\pi}{3}$$

Also $(1-i\sqrt{3})^7 = \left(2e^{-i\frac{\pi}{3}}\right)^7 = 2^7 e^{-i\frac{7\pi}{3}} = \underline{2^7 e^{-i\frac{\pi}{3}}} = \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) 2^7$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$ (geometrisch, Zettel 1)

$\sin\left(-\frac{\pi}{3}\right) = \pm\sqrt{1-\cos^2\left(-\frac{\pi}{3}\right)} = \pm\sqrt{1-\frac{1}{4}} = \pm\frac{\sqrt{3}}{2}$

wähle neg. Vorz., da $\sin(x)<0$ für $x \in (-\pi, 0)$

c) $\frac{(1+i)^{14}}{(\sqrt{3}-i)^7} = \left(\frac{(1+i)^2}{(\sqrt{3}-i)}\right)^7 = \left(\frac{2i}{\sqrt{3}-i}\right)^7 = \left(\frac{2i(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}\right)^7 = \left(\frac{2\sqrt{3}i-2}{3+1}\right)^7$

$= \left(-\frac{1}{2}\right)^7 \left(1-\sqrt{3}i\right)^7 \overset{b)}{=} -\frac{1}{2^7}\cdot 2^7 e^{-i\frac{\pi}{3}} = -e^{-i\frac{\pi}{3}} = \left(\underbrace{e^{i\pi}}_{=-1}\right)\left(e^{-i\frac{\pi}{3}}\right) = \underline{e^{i\frac{2\pi}{3}}} = \underline{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$ (siehe b))

A2. a) $\mathrm{Re}(w) = \sqrt{3}\,\mathrm{Im}(w)$, $|w|=3$, $\arg(w)<0$. Sei $w=x+iy$.

$$\left.\begin{array}{l} |w|=3 \Rightarrow x^2+y^2=9 \\ \mathrm{Re}(w)=\sqrt{3}\,\mathrm{Im}(w) \Rightarrow x=\sqrt{3}y \end{array}\right\} \Rightarrow (\sqrt{3}y)^2+y^2=9 \Rightarrow 3y^2+y^2=9 \Rightarrow 4y^2=9$$

$$\Rightarrow y^2 = \frac{9}{4} \Rightarrow y = \pm\frac{3}{2}$$

$\arg(w)<0 \Rightarrow y = -\frac{3}{2}$.

Also $w = \underline{\underline{\frac{3\sqrt{3}}{2} - \frac{3}{2}i}} = \frac{\sqrt{27}}{2} - \frac{3}{2}i$

b) $i\cdot z = \overline{iz}$, $z\cdot\bar{z}=8$, $\mathrm{Im}(z)<0$. Sei $z=x+iy$.

$iz = \overline{iz} \Rightarrow iz = -i\bar{z} \Rightarrow z = -\bar{z} \Rightarrow x+iy = -x+iy \Rightarrow x=-x \Rightarrow x=0$

$z\cdot\bar{z} = 8 \Rightarrow (0+iy)(0-iy) = y^2 = 8 \Rightarrow y = \pm\sqrt{8}$

$\mathrm{Im}(z)<0 \Rightarrow y = -\sqrt{8}$

Also $\underline{\underline{z = -\sqrt{8}\,i}}$.

3te Einheitswurzeln $\zeta_1 = e^{i0} = 1$, $\zeta_2 = e^{\frac{2\pi}{3}}$, $\zeta_3 = e^{\frac{4\pi}{3}}$

A3. $\sqrt[3]{4-4\sqrt{3}i} \overset{\circ}{=} \sqrt[3]{8e^{-i\pi/3}} = \sqrt[3]{8}\,\sqrt[3]{e^{-i\pi/3}} = 2\sqrt[3]{e^{-i\pi/3}} = \left\{2e^{-i\frac{\pi}{3}\cdot\frac{1}{3}}, 2e^{-i\frac{\pi}{3}\cdot\frac{1}{3}+\frac{2\pi}{3}}, 2e^{-i\frac{\pi}{3}\cdot\frac{1}{3}+\frac{4\pi}{3}}\right\}$

$r = \sqrt{4^2+(4\sqrt{3})^2} = \sqrt{16+16\cdot 3} = \sqrt{4\cdot 16} = 2\cdot 4 = 8$

$\varphi = \arccos\left(\frac{4}{8}\right) = -\arccos\left(\frac{1}{2}\right) = -\frac{\pi}{3}$

$$= \underline{\underline{\left\{2e^{-\frac{\pi}{9}i}, 2e^{\frac{5}{9}\pi i}, 2e^{\frac{11\pi}{9}i}\right\}}}$$

A4 $s(t) = s_1(t) + s_2(t)$, $s_1(t) = 2\cos(2t - \frac{\pi}{6})$, $s_2(t) = 4\cos(2t + \frac{\pi}{2})$

In der Notation von Skript Seite 19 ist $A_1 = 2$, $A_2 = 4$, $\omega = 2$, $\alpha_1 = -\frac{\pi}{6}$, $\alpha_2 = \frac{\pi}{2}$,

also $a_1 = 2e^{-i\frac{\pi}{6}}$, $a_2 = 4e^{i\frac{\pi}{2}} = 4i$, $A = |a_1 + a_2| = \ldots$, $\varphi = \arg(a_1 + a_2)$

Berechne kart. Darst. von $a_1 = 2e^{-i\frac{\pi}{6}} = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i$

$$A = |a_1 + a_2| = |\sqrt{3} - i + 4i| = |\sqrt{3} + 3i| = \sqrt{3 + 9} = \sqrt{12} = 2\sqrt{3}$$

$$\varphi = \arg(\sqrt{3} + 3i) = \arccos\left(\frac{\sqrt{3}}{\sqrt{3+9}}\right) = \arccos\left(\frac{\sqrt{3}}{\sqrt{12}}\right) = \arccos\left(\sqrt{\frac{1}{4}}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Also

$$s(t) = A \cdot \cos(\omega t + \varphi) = 2\sqrt{3}\cos\left(2t + \frac{\pi}{3}\right).$$

A5 a) $q(x) = p_1(x) + p_2(x) = x^4 + x^2 + 1 + x^2 + x + 1 = x^4 + \underbrace{(1+1)}_{=0}x^2 + x + \underbrace{1+1}_{=0} = x^4 + x$

$r(x) = p_1(x) \cdot p_2(x) = (x^4 + x^3 + 1)(x^2 + x + 1) = x^6 + x^5 + \underbrace{x^4 + x^4}_{=0} + x^3 + \underbrace{x^2 + x^2}_{=0} + x + 1 = x^6 + x^5 + x^3 + x + 1$

$s(x) = p_2(x) \cdot p_3(x) = (x^2 + x + 1)(x^3 + x) = \ldots = x^5 + x^4 + x^2 + x$

b) $q(x)$ und $s(x)$ beschreiben dieselbe Polynomfkt auf $\mathbb{B}$, nämlich $\begin{matrix} 0 \mapsto 0 \\ 1 \mapsto 0 \end{matrix}$, also die Nullfkt.

c)

$0 \mapsto 0$, $1 \mapsto 0$ $f_1(x) = 0$	$0 \mapsto 0$, $1 \mapsto 1$ $f_2(x) = x$
$0 \mapsto 1$, $1 \mapsto 1$ $f_3(x) = 1$	$0 \mapsto 1$, $1 \mapsto 0$ $f_4(x) = x + 1$