12. Assignment on

Computational Geometry

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Due on 1. July 2013 in class

Problem 1 Summation by Parts *10 points*

In class, we used summation by parts in order to bound the expected conflict change. Let $a_1, \dots, a_{m+1}, b_1, \dots, b_{m+1}$ two sequences of real numbers. Show that

$$\sum_{i=1}^{m} a_i(b_{i+1} - b_i) = a_{m+1}b_{m+1} - a_1 b_1 - \sum_{i=1}^{m} b_{i+1}(a_{i+1} - a_i).$$

Conclude that

$$\sum_{k=1}^{n-4} \frac{6}{k^2}\Big(|L_{\le k}| - |L_{\le (k-1)}|\Big) = \frac{6}{(n-3)^2}|L_{\le (n-4)}| - 6|L_0| + \sum_{k=1}^{n-4} |L_{\le k}|\Big(\frac{6}{k^2} - \frac{6}{(k+1)^2}\Big).$$

Problem 2 Point in Polytope *10 points*

Let $\mathcal{P} \subseteq \mathbb{R}^3$ be a simplicial (convex) polytope, represented as a DCEL. Describe a data structure for the following queries: given a point $q \in \mathbb{R}^3$, does q lie inside $\mathcal{P}$? Your data structure should be able to answer a query in $O(\log n)$ expected time. Analyze the preprocessing time and space requirement of your structure.

Hint: Use a data structure for planar point location.

Problem 3 Chew's Algorithm *10 points*

In this problem we will consider Chew's algorithm, a randomized algorithm for computing the Voronoi diagram of the vertices of a convex polygon. Let $\mathcal{P}$ be a convex polygon with n vertices. Let S be the vertex set of $\mathcal{P}$. Chew's Algorithm works as follows:

(1) Pick a random point $s \in S$ and delete s from $\mathcal{P}$. This gives a new convex polygon $\mathcal{P}'$ with vertex set $S \setminus \{s\}$.

(2) Recursively compute the Voronoi diagram $\mathcal{V}(S \setminus \{s\})$ for the vertices of $\mathcal{P}'$.

(3) Let q and r be the neighbors of s on $\partial\mathcal{P}$. Starting from the Voronoi vertex of $\mathcal{V}(S \setminus \{s\})$ incident to the Voronoi edge for q and r, determine the set D of Voronoi vertices of $\mathcal{V}(S \setminus \{s\})$ that are in conflict with s, i.e., that are not Voronoi vertices of $\mathcal{V}(S)$. The set D is connected in $\mathcal{V}(S \setminus \{s\})$. Delete D from $\mathcal{V}(S \setminus \{s\})$ and create appropriate Voronoi vertices for s. This gives $\mathcal{V}(S)$.

Do the following:

(a) Illustrate Step (3) through a helpful drawing.

(b) Briefly justify the claims made in Step (3).

(c) Show how to implement Step (3) in time $O(d)$, where d is the number of Voronoi vertices on the Voronoi cell for s.

(d) Use backwards analysis to show that the expected work in Step (3) is constant.

(e) Conclude that Chew's algorithm requires $O(n)$ expected time.

(f) Why is it important that $\mathcal{P}$ is convex? Can you say *in one sentence*, why Chew's algorithm requires only linear time, although in general we need time $\Theta(n \log n)$ to find a Voronoi diagram?