1. Assignment on

Computational Geometry

SoSe 2013

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Problem 1 Computing the Minimum *10 points*

Consider the following algorithm for computing the minimum in an array `A` of n distinct numbers:

```
curMin := A[1]
for i := 2 to n do
  if A[i] < curMin then
    curMin := A[i]  // (*)
return curMin
```

Our goal is to compute the expected number of times that line (*) is executed, assuming that the order of elements in `A` is a random permutation. Let X be the random variable that represents how often line (*) is encountered.

(a) Let $X_i \in \{0, 1\}$ be the random variable that indicates whether (*) is executed in the ith iteration of the `for`-loop. Show that $\mathrm{E}[X] = \sum_{i=2}^{n} \mathrm{E}[X_i]$.

(b) Find $\mathrm{E}[X_i]$.
Hint: What is the probability that `A`$[i]$ is the smallest element in a random permutation of i elements?

(c) Conclude that $\mathrm{E}[X] = O(\log n)$.

Problem 2 Induction *10 Punkte*

Let L be a set of n lines in the plane. The regions that are bounded be the lines in L are called *faces*. Two faces are *adjacent* if they share a common part of a line on their boundary.

We would like to assign a color to each face such that no two adjacent faces have the same color. Show that two colors are always sufficient.

Problem 3 O-Notation *10 Punkte*

Prove or disprove:

(a) $\log n! = \Theta(n \log(n))$;

(b) $\log(mn) = O(\log(n + m))$;

(c) let $f, g : \mathbb{N} \to \mathbb{N}$ such that $f(n), g(n) > 2$ and $f(n) = O(g(n))$:
  (i) $\sqrt{f(n)} = O(\sqrt{g(n)})$;
  (ii) $2^{f(n)} = O(2^{g(n)})$.