## Discrete Geometry, SS 2011 - exercise sheet 7

due date: Tuesday, June 7th, 2011, 14:00
30. An afterthought on the balanced case of the Szemerédi-Trotter bound (0 credits)
(a) Prove the incidence bounds $I(m, n) \leq \min \left\{n^{2}+m, m^{2}+n\right\}$.
(b) Show that the proof of the balanced case of the Szemerédi-Trotter bound $I(n, n)=$ $O\left(n^{4 / 3}\right)$ via $\frac{1}{r}$-cuttings goes through even with the "very weak" incidence bounds from part (a).
(c) Does the proof still extend to the unbalanced case, as in Exercise 21?
31. Clarkson's bound on levels. (0 credits)

Consider an arrangement of $n$ lines in general position.
(a) Give an upper bound on the number of vertices of level 0 .
(b) Consider a vertex $v$ at level $\ell$ of the arrangement and a subset $R$ of the lines. Which lines must (must not) belong to $R$ in order that $v$ is a vertex of the 0-level of $R$ ?
(c) In the lecture, we have computed the probalility that $v$ is a vertex of the 0 -level of $R$, if each line is chosen independently as an element of $R$ with probability $p$. What is the probalility that $v$ is a vertex of the 0 -level of $R$, if $R$ is uniformly chosen among all subsets with a given cardinality $r$ ?
(d) Show that this probability is a decreasing function of $\ell$.
32. Extreme vertices in simple arrangements (10 credits)
(a) Consider $n$ lines in the plane in general position. A vertex $v$ of the arrangement that is an intersection between a line of positive slope and a line of negative slope is called an extreme vertex. By imitating Clarkon's proof on $(\leq k)$-levels, show that there are at most $O\left((k+1)^{2}\right)$ extreme vertices of level at most $k$.
(b) Show that this bound cannot be improved.
(c) (0 credits) Why are these vertices called extreme?
33. The zone theorem ( 10 credits)

Consider an arrangement $A$ of $n$ lines in the plane and another line $g$, distinct from the lines of $A$. The zone of $g$ is the set of (closed) cells that are intersected or touched by $g$. Prove that the zone of a line contains in total $O(n)$ vertices and edges.
Hint: Use the bound on Davenport-Schinzel sequences of order 2 from exercise 29a. You may need to distinguish between the two sides of each line in $A$, and the two sides of $g$.
34. Visibility polygons, lower envelopes of line segments (10 credits)

Consider an arrangement of $n$ line segments in the plane, not necessarily in general position, and a point $p$ not on any of these segments. The visibility polgon of $p$ is the closure of the union of all segments that have an endpoint in $p$ and are disjoint from the given segments.
Give an upper bound on the number of vertices of the visibility polygon in terms of Davenport-Schinzel sequences of order $s$, for an appropriate vales of $s$.

