due date: Tuesday, May 10th, 2011, 14:00

18. The Crossing Number Theorem in multigraphs (10 credits)

Show that the Crossing Number Theorem holds for multigraphs G = (V, E) in which the multiplicity of every edge is bounded by k:

$$\# \text{crossings} \ge a_k \cdot \frac{|E|^3}{|V|^2} \text{ if } |E| \ge b_k |V|,$$

for suitable constants a_k, b_k depending on k.

Try to find good upper and lower bounds for the possible values of a_k and b_k .

19. Shattered point sets (10 credits)

A point set S is *shattered* by a family \mathcal{R} of *ranges* if every subset of S can be "cut out" from S by a range in \mathcal{R} :

$$\forall S' \subseteq S \colon \exists R \in \mathcal{R} \colon S \cup R = S'$$

What is the largest set of points in the plane that is shattered by

- (a) circular disks?
- (b) half-planes?
- (c) convex sets?
- (d) triangles?
- (e) vertical strips? ("rectangles" that are unbounded vertically)
- (f) unions of two vertical strips?

20. Shatter functions (10 credits)

How many point sets can one cut out of a set of n points in the plane

- (a) with circular disks?
- (b) with half-planes?
- (c) with convex sets?
- (d) with axis-parallel squares?
- (e) with axis-parallel rectangles?
- (f) with arbitrary rectangles?

Determine the maximum possible value asymptotically in terms of n.