## Discrete Geometry, SS 2011 - exercise sheet 4

due date: Tuesday, May 10th, 2011, 14:00

18. The Crossing Number Theorem in multigraphs (10 credits)

Show that the Crossing Number Theorem holds for multigraphs $G=(V, E)$ in which the multiplicity of every edge is bounded by $k$ :

$$
\# \text { crossings } \geq a_{k} \cdot \frac{|E|^{3}}{|V|^{2}} \text { if }|E| \geq b_{k}|V|
$$

for suitable constants $a_{k}, b_{k}$ depending on $k$.
Try to find good upper and lower bounds for the possible values of $a_{k}$ and $b_{k}$.
19. Shattered point sets (10 credits)

A point set $S$ is shattered by a family $\mathcal{R}$ of ranges if every subset of $S$ can be "cut out" from $S$ by a range in $\mathcal{R}$ :

$$
\forall S^{\prime} \subseteq S: \exists R \in \mathcal{R}: S \cup R=S^{\prime}
$$

What is the largest set of points in the plane that is shattered by
(a) circular disks?
(b) half-planes?
(c) convex sets?
(d) triangles?
(e) vertical strips? ("rectangles" that are unbounded vertically)
(f) unions of two vertical strips?
20. Shatter functions ( 10 credits)

How many point sets can one cut out of a set of $n$ points in the plane
(a) with circular disks?
(b) with half-planes?
(c) with convex sets?
(d) with axis-parallel squares?
(e) with axis-parallel rectangles?
(f) with arbitrary rectangles?

Determine the maximum possible value asymptotically in terms of $n$.

