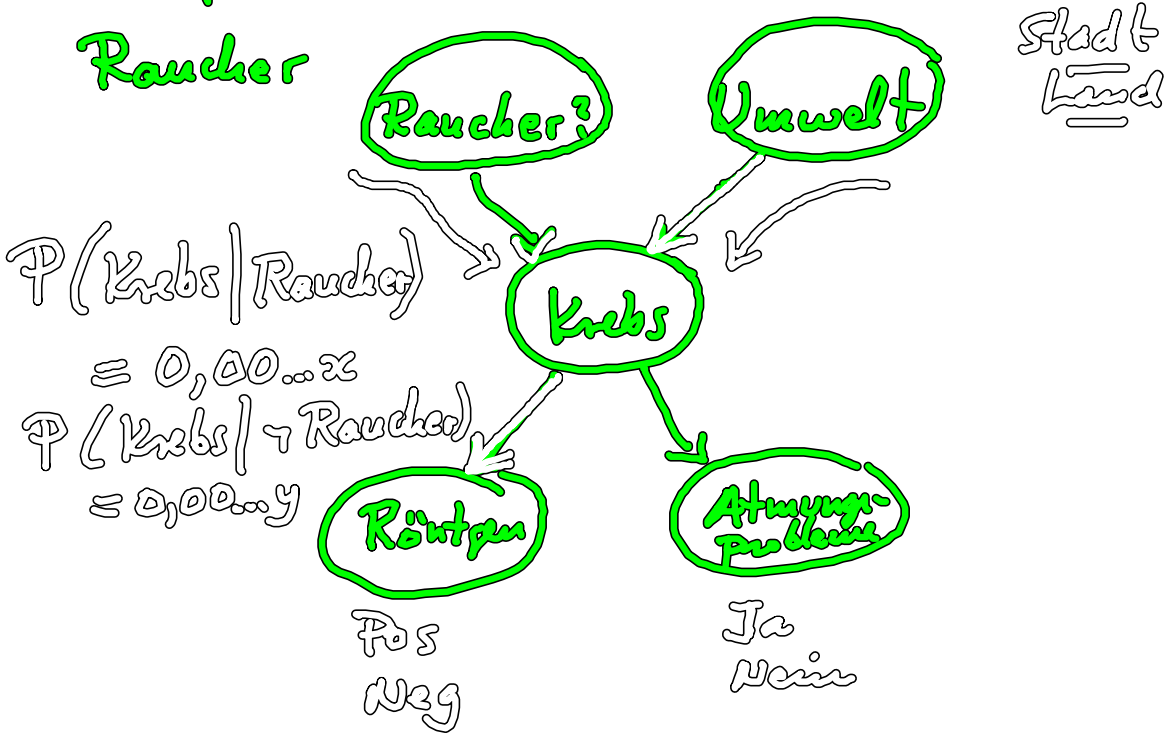
Bayes-Netze

$A = \{ \begin{matrix} T & 1 \\ F & 0 \end{matrix}$ Wahrheitswert 0..1

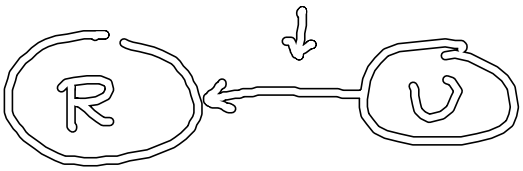
0,5
0,2
0,3
Fuzzy-Logik

Wahrscheinlichkeiten

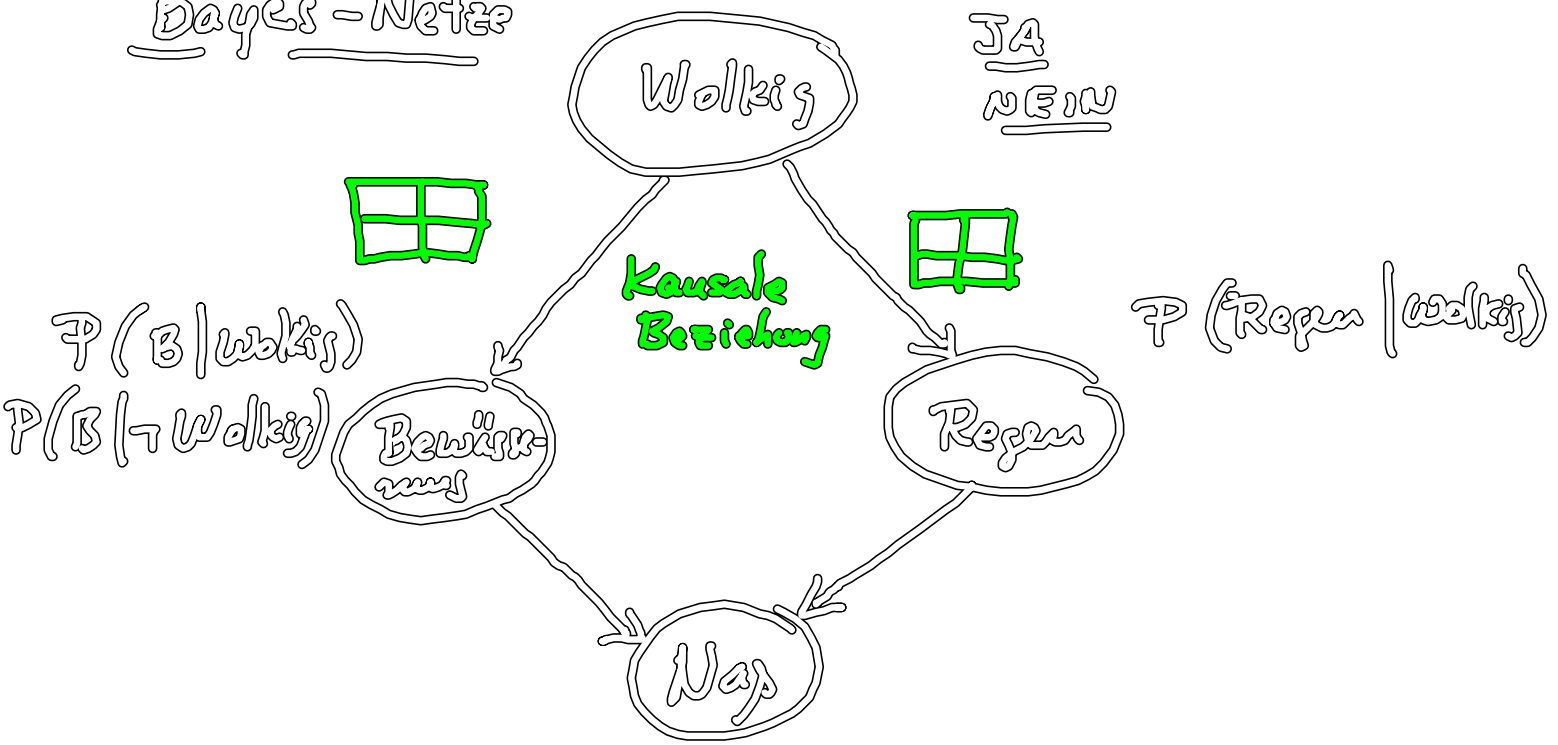
Beispiel:
Raucher



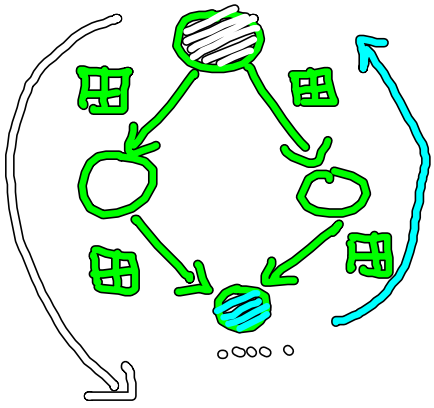
Kein Pfeil ⇒ Knoten sind unabhängig



Bayes-Netze



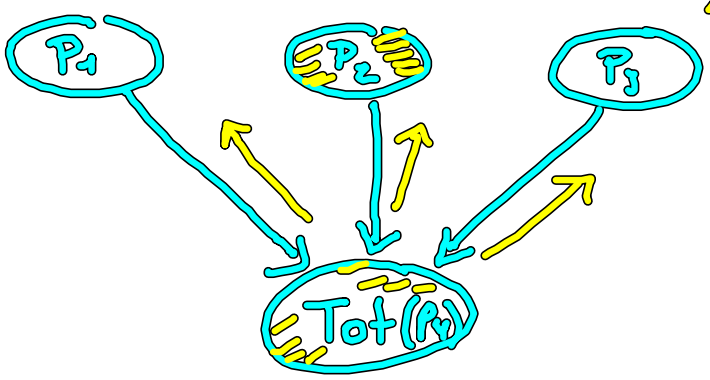
Statistik



← Unabhängigkeit

$P(Nass \mid Wolkig)$?

$P(Wolkig \mid Nass)$? Inferenz



Evidenz

$P(\text{Mord}|\text{Waffe}) = 0,8$

$P(\text{Mörder}|\neg\text{Waffe}) = 0,2$

Bayes-Formel

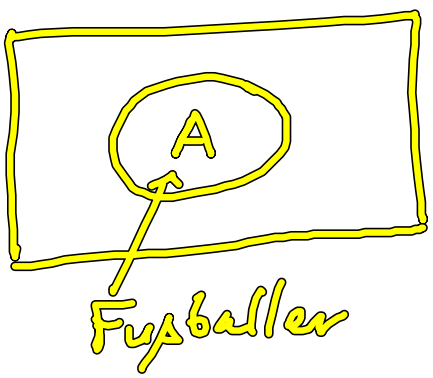
Maß P Universum Ω $\emptyset$

$P(\Omega) = 1$

$P(\emptyset) = 0$

Ereignisse $A \subset \Omega$

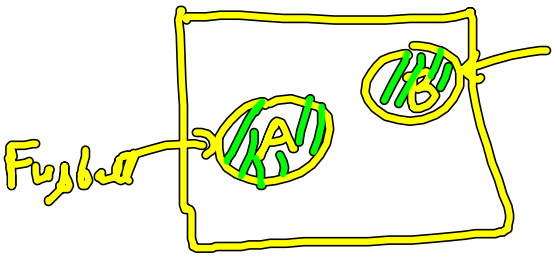
Ω = Personen



$P(A) = 0,05$

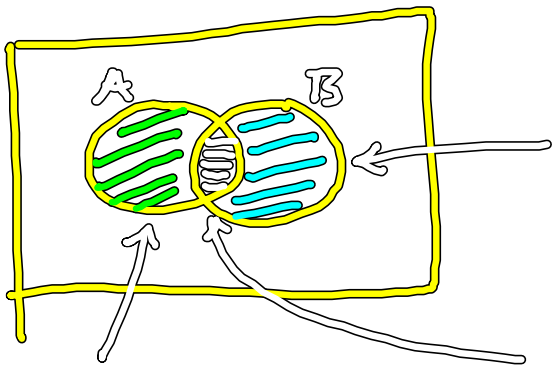
$$P(A \cup B) = P(A) + P(B)$$

$$A \cap B = \emptyset$$



$P(A \cup B)$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \cap B)$$



$B - (A \cap B)$

$A \cap B$

$A - (A \cap B)$

$$P(A \text{ or } B) = P(A \cap B^c) + P(B \cap A^c) + P(A \cap B)$$

$$P(A \text{ or } B) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$+ P(A \cap B)$$

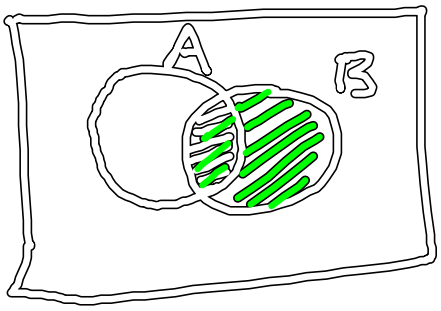
$$= P(A) + P(B) - P(A \cap B)$$

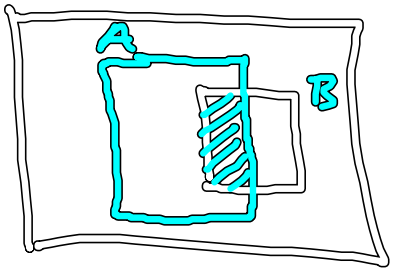
Def:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A|B)$: Kopf-schmerzen | B: Grippe

$$P(A|B) = \frac{1}{2}$$





$$\Rightarrow \quad P(A \cap B) = P(A|B)P(B)$$

$$P(A \cap B) = P(B|A)P(A)$$

$$\Rightarrow \quad P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

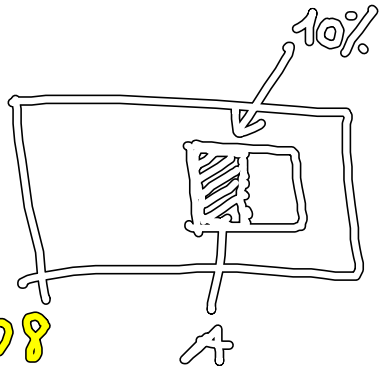
Bayes Formel

Gegeben: Skipper

$P(B) = 0{,}1$

Kopfs. $P(A) = 0{,}05$ $\underline{0{,}08}$

$P(A|B) = 0{,}5$



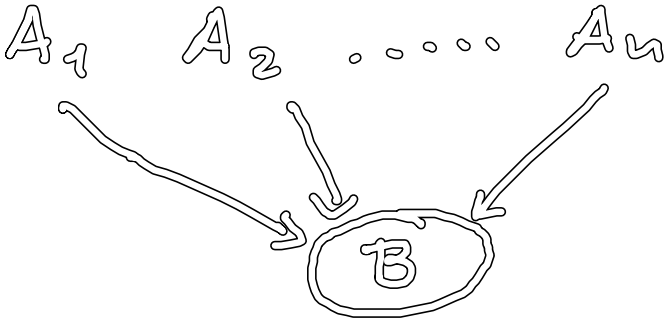
$$P(B|A) = \frac{0{,}5 \cdot 0{,}1}{0{,}05}$$

$=$

$$= \frac{(0{,}5)(0{,}1)}{0{,}08}$$

$\approx \underline{0{,}6}$

Verallgemeinerte Bayes-Regel



$P(B|A_1)$ $P(B|A_2)$ .....

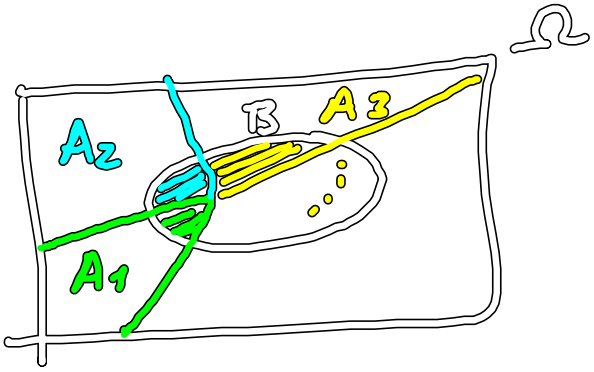
$P(B)$

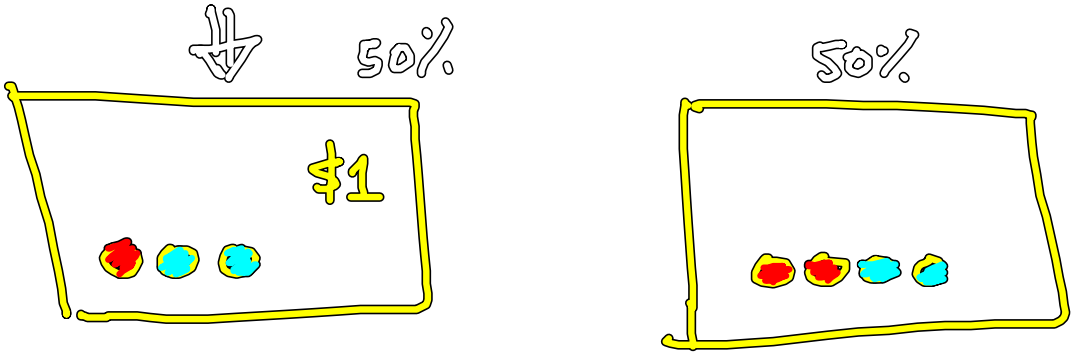
Verallgemeinerte Bayes

$$P(A_1|B) = \frac{P(B|A_1)\,P(A_1)}{P(B)}$$

$$= \frac{P(B|A_1)\,P(A_1)}{\sum_{i=1}^{n} P(B|A_i)\,P(A_i)}$$

A_i's unabhängig!





E-Boy: Was viel soll ich zahlen?

50 cent

Schnipsel ist rot
wieviel soll ich schätzen?

Formalisieren

U_1 #1 U_2

$P(\text{rot} \mid U_1) = 1/3$ $\quad P(\text{Gewinn} \mid U_1) = 1$

$P(\text{rot} \mid U_2) = 1/2$ $\quad P(\text{Gewinn} \mid U_2) = 0$

$P(\text{blau} \mid U_1) = 2/3$ $\quad P(U_1) = 1/2$

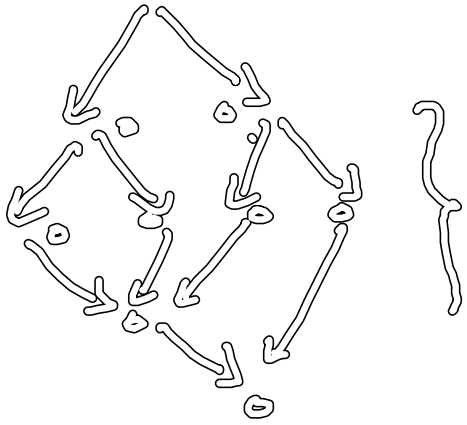
$P(\text{blau} \mid U_2) = 1/2$ $\quad P(U_2) = 1/2$

$$P(U_1 \mid \text{rot}) = \frac{P(\text{rot} \mid U_1)\, P(U_1)}{P(\text{rot})}$$

$$= \frac{P(\text{rot} \mid U_1)\, P(U_1)}{P(\text{rot} \mid U_1)\, P(U_1) + P(\text{rot} \mid U_2)\, P(U_2)}$$

$$= \frac{(1/3)(1/2)}{\frac{1}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}}$$

$$= \frac{1/3}{5/6} = \frac{2}{5} = 0{,}4$$



Personen P_1 P_2 P_3

A-priori Verdacht

$$\pi(x) = \begin{pmatrix} 0,8 \\ 0,1 \\ 0,1 \end{pmatrix}$$

$P(\text{Mörder} \mid \text{hatte Waffe getötet}) = 0,8$

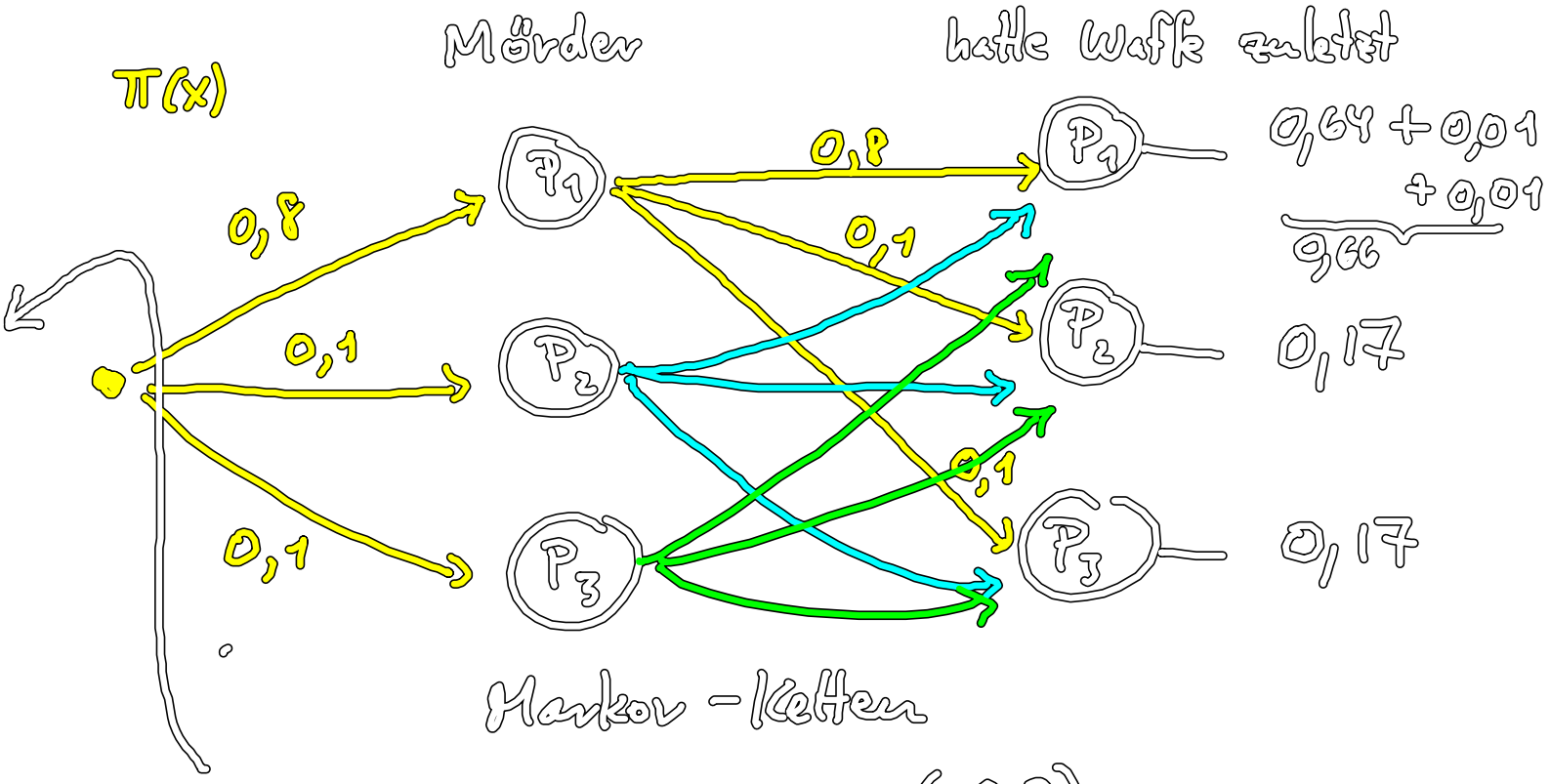
$P(\text{Mörder} \mid \neg \quad " \quad) = 0,2$

	x Mörder				Evidenz		
	$\pi(x)$	Belief	Evidenz-Info		$\pi(y)$	Belief	Labor
P_1	0,8	0,8	1	μ	0,66	0,66	1
P_2	0,1	0,1	1		0,17	0,17	1
P_3	0,1	0,1	1		0,17	0,17	1
		1					

P_1 hatte Waffe getötet

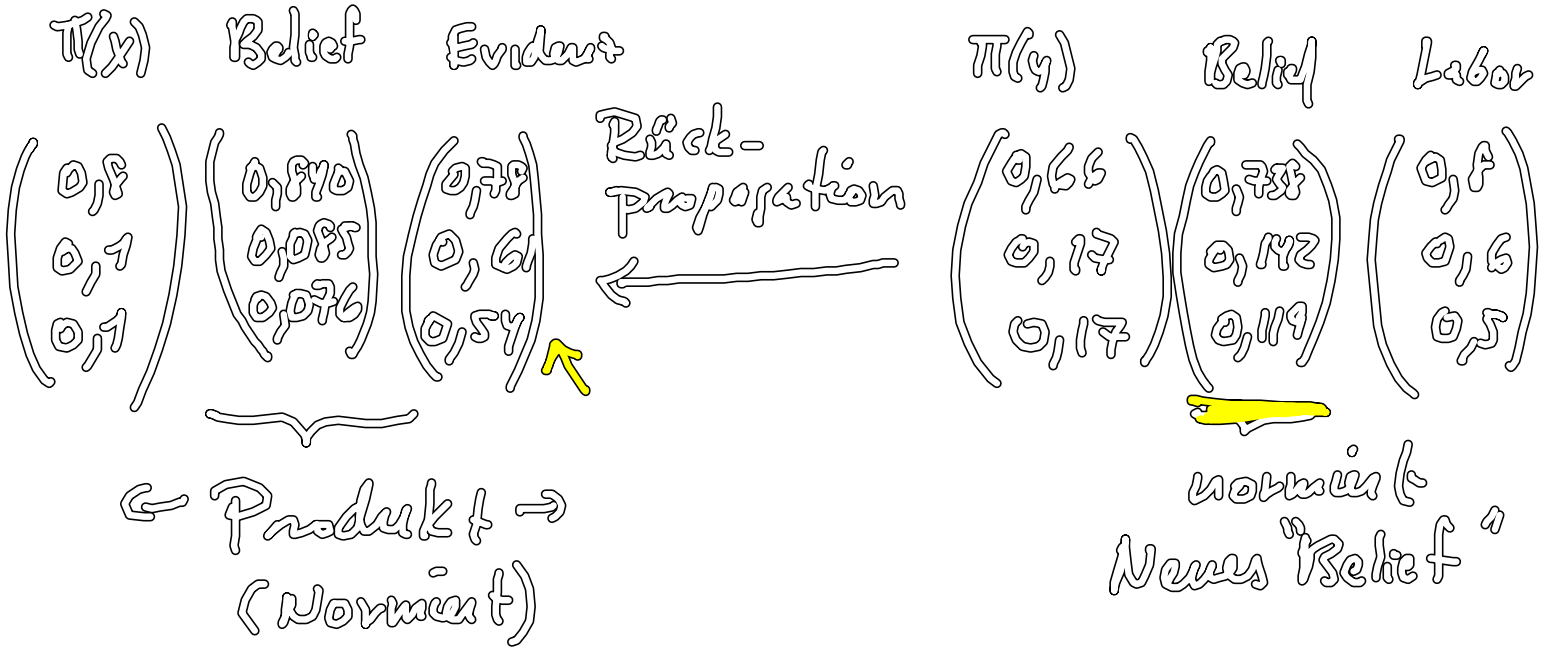
Mörder Person

$$M = \begin{pmatrix} 0{,}8 & 0{,}1 & 0{,}1 \\ 0{,}1 & 0{,}8 & 0{,}1 \\ 0{,}1 & 0{,}1 & 0{,}8 \end{pmatrix}$$



Markov-Ketten

$$\text{Laborbericht} = \begin{pmatrix} 0{,}8 \\ 0{,}6 \\ 0{,}5 \end{pmatrix}$$

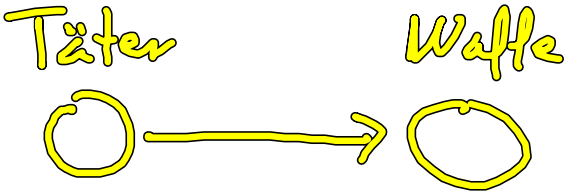


$$M\begin{pmatrix}0,8\\0,6\\0,5\end{pmatrix}$$

$$\begin{pmatrix}0,8 & 0,1 & 0,1\\0,1 & 0,8 & 0,1\\0,1 & 0,1 & 0,8\end{pmatrix}\begin{pmatrix}0,8\\0,6\\0,5\end{pmatrix}$$

$$\pi(x) = \begin{pmatrix}0,28\\0,36\\0,36\end{pmatrix}$$

π(x	Belief	
0,28	0,343	0,75
0,36	0,349	0,61
0,36	0,308	0,54

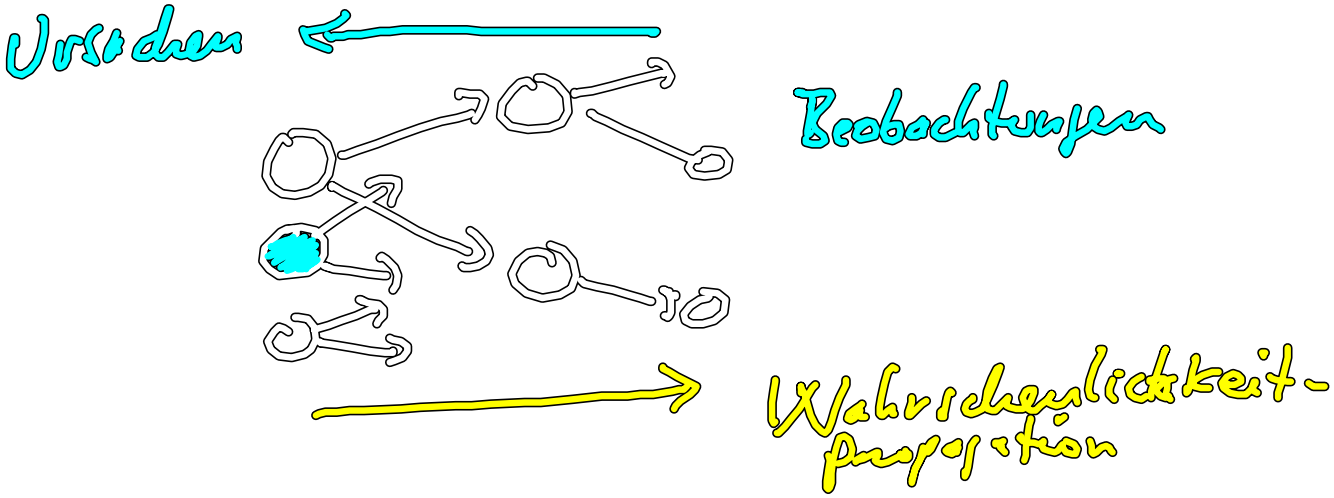


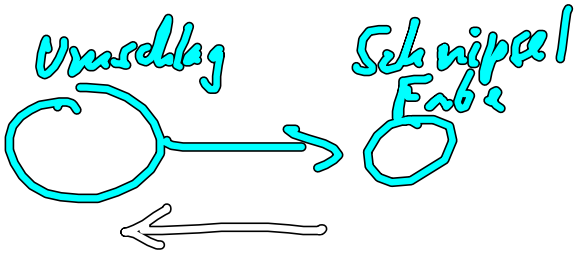
Judea Pearl

Bayes Netze = Graphische Modelle

Ereignisse = Knoten

Kanten = Kausalität





Einfach

Bayes – Wahrscheinlichkeit – Rückpropagation

⇒ NP schwer

KI