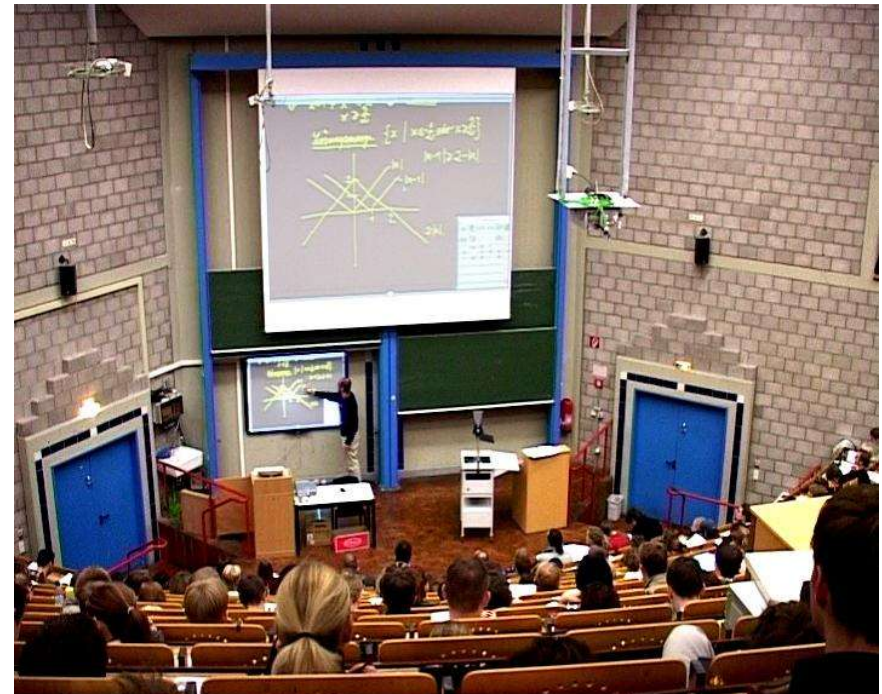


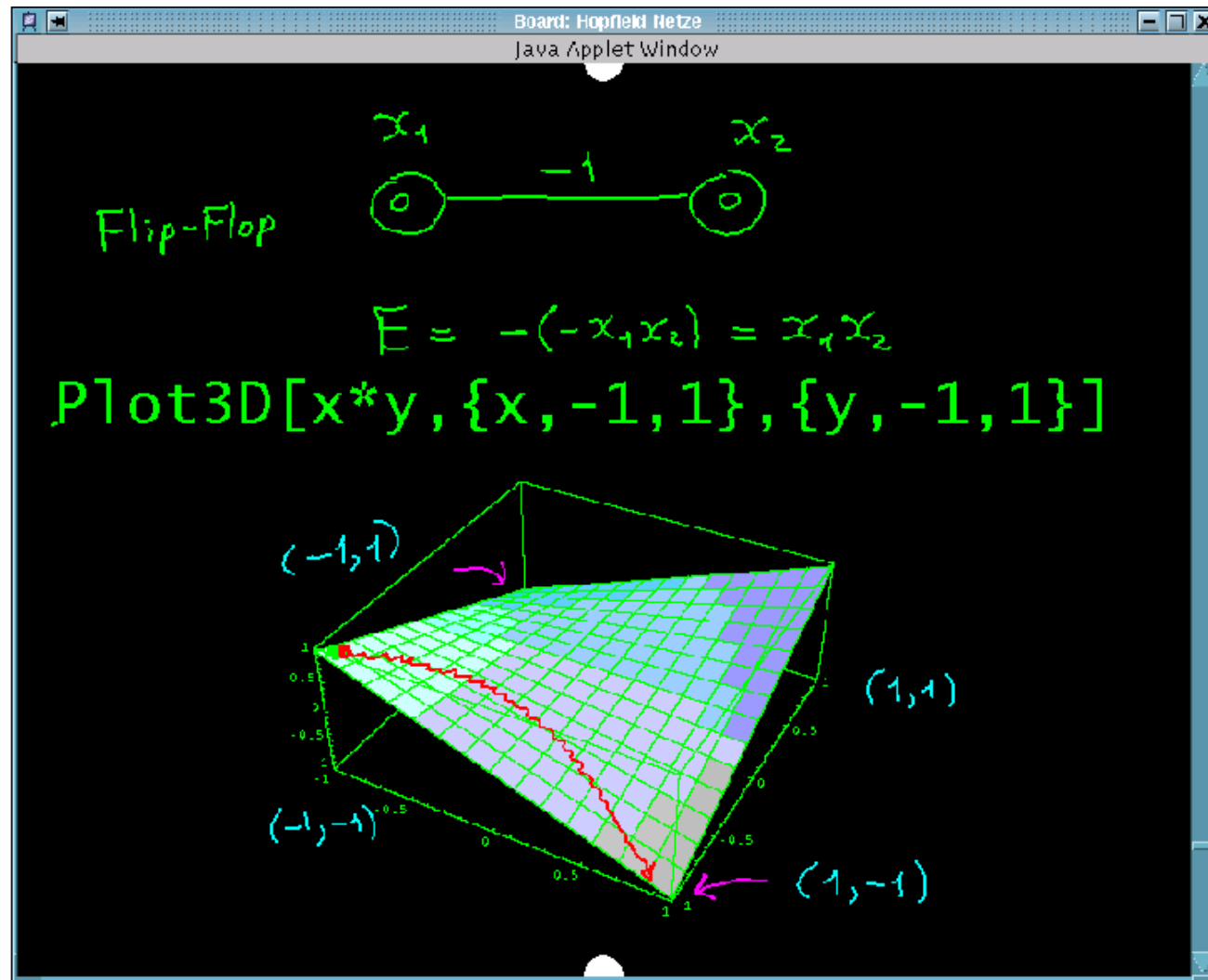
On-Line Erkennung handgeschriebener mathematischer Formeln

Ernesto Tapia
Berlin 9. Juni 2004

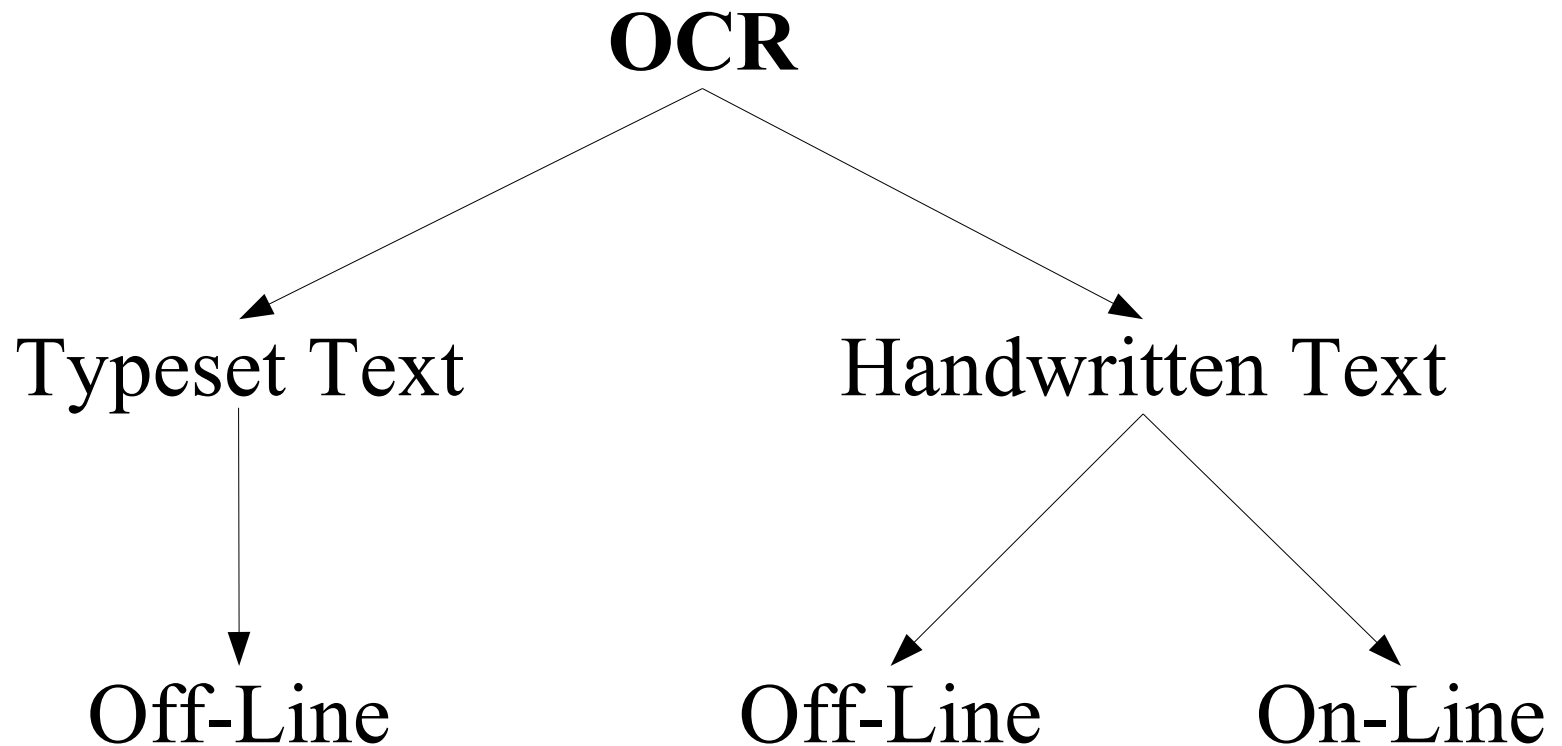
Motivation: The electronic Chalk Board (E-Chalk Board)



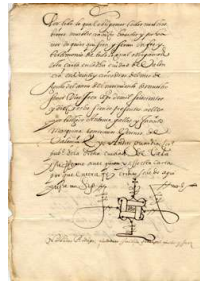
Motivation: The electronic Chalk Board (E-Chalk)



Optical Character Recognition



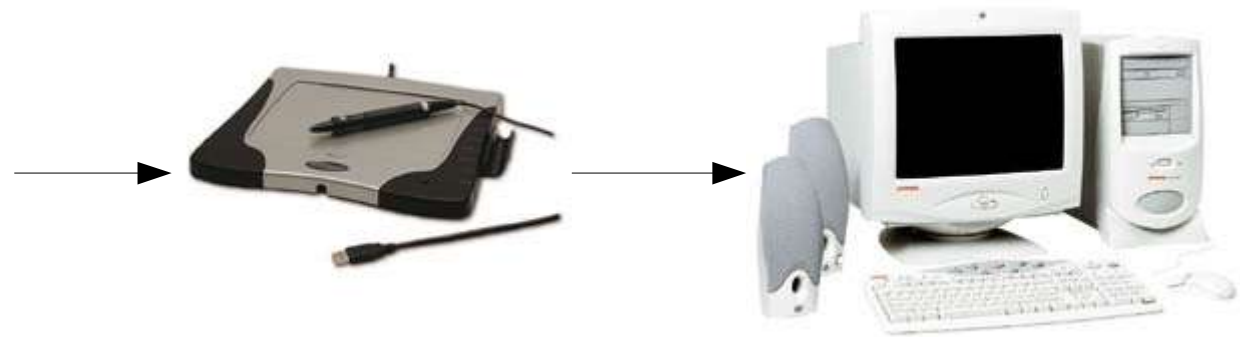
Optical Character Recognition: Off-line Data



$$\frac{1}{2\pi i} \int_C \Lambda(z) dz = \sum_{\mu=1}^m \operatorname{Res}_{\mu} \Lambda(z)$$

$$\frac{1}{2\pi i} \int_C f(z) dz = \sum_{\mu=1}^m \operatorname{Res}_{\mu} f(z)$$

Optical Character Recognition: On-line Data



$$\frac{1}{2\pi i} \int_C f(z) dz = \sum_{\mu=1}^m \text{Res}_{z_\mu} f(z)$$

On-line Handwriting: Styles

BOXED

SPACED

run-on

cursive

Recognition of mathematical Expressions

Characteristics:

- Scope of recognition Systems

- Grouping of symbols

$$\mathit{ sint } \sim \sin t$$

- Explicit and implicit operators

$$a + b, a^b, ab.$$

- Ambiguity of symbols

$$\Sigma \text{ is sigma or sum. } dy, cx + dy, \int y dy$$

- Irregular writing

$$2^x \ 2^x \ 2^x \ 2^x \ 2x \ 2x \ 2x \ 2x$$

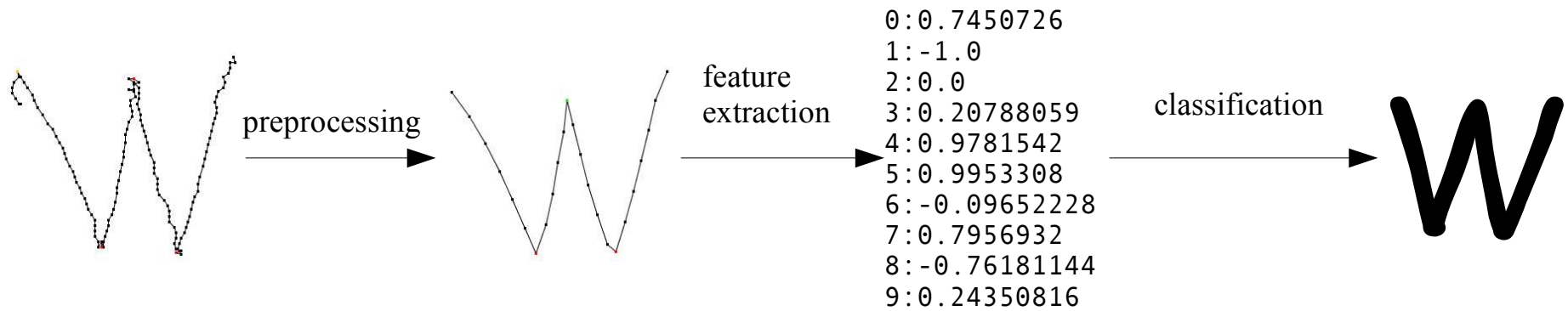
Recognition of mathematical Expressions

A two-step approach:

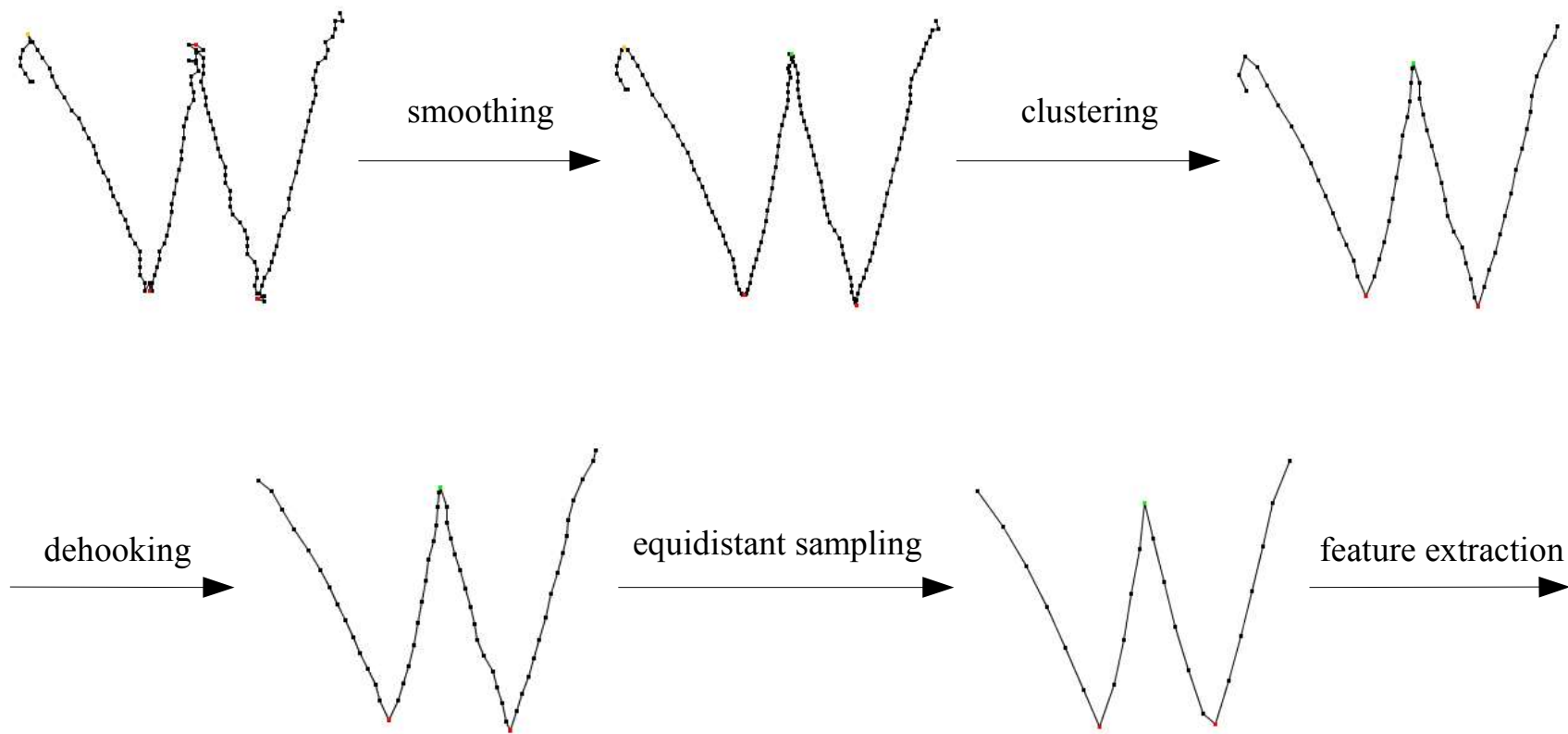
- 1) Symbol recognition

- 2) Structural analysis of the expression

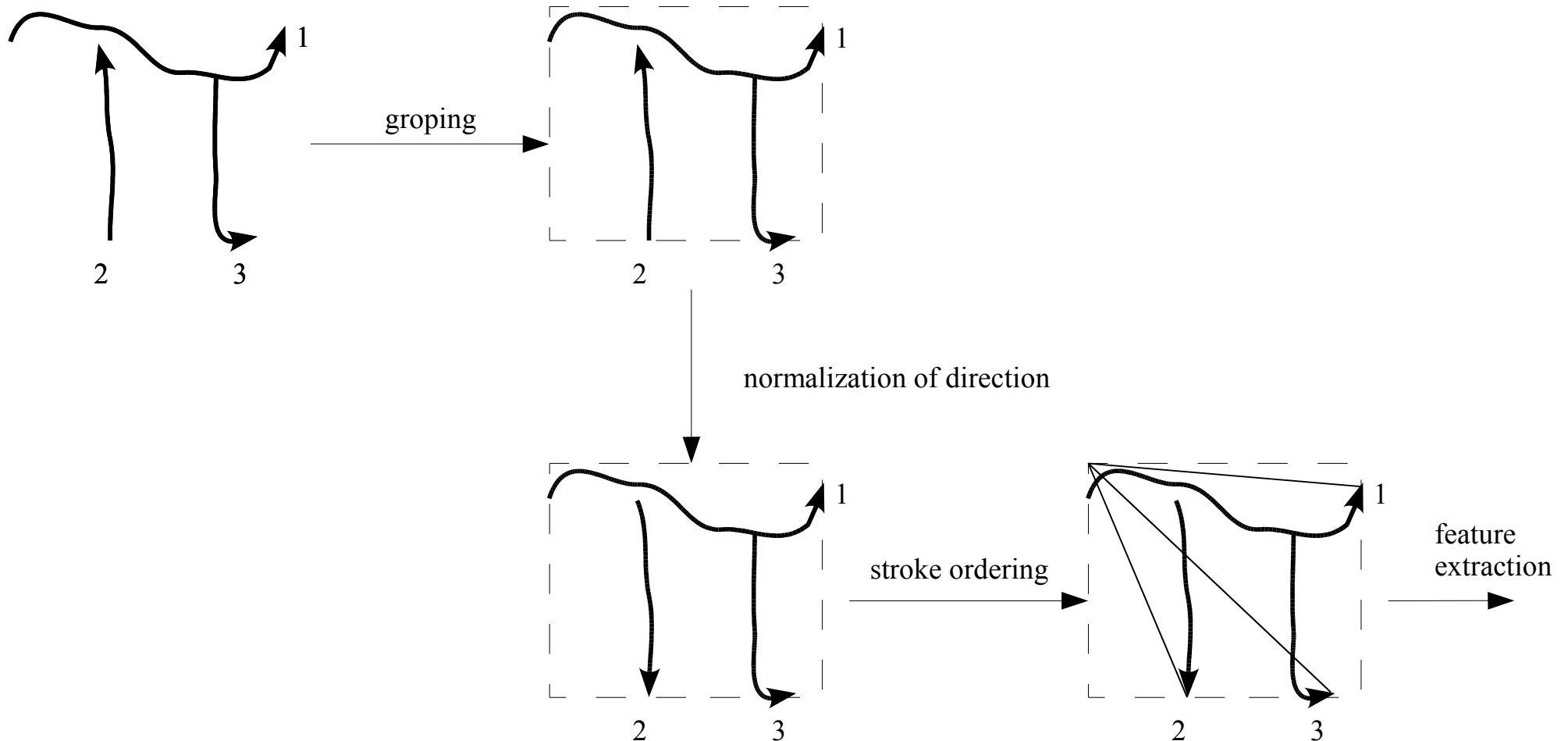
Symbol Recognition: Classification



Symbol Recognition: Preprocessing



Symbol Recognition: Preprocessing Symbols with several Strokes

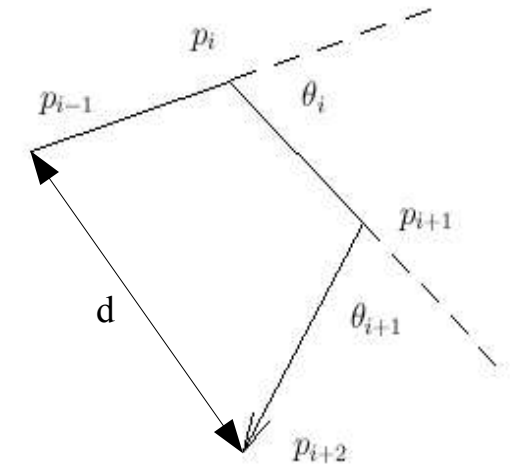


Symbol Recognition: Features

Given a stroke (p_1, p_2, \dots, p_n)

Local features:

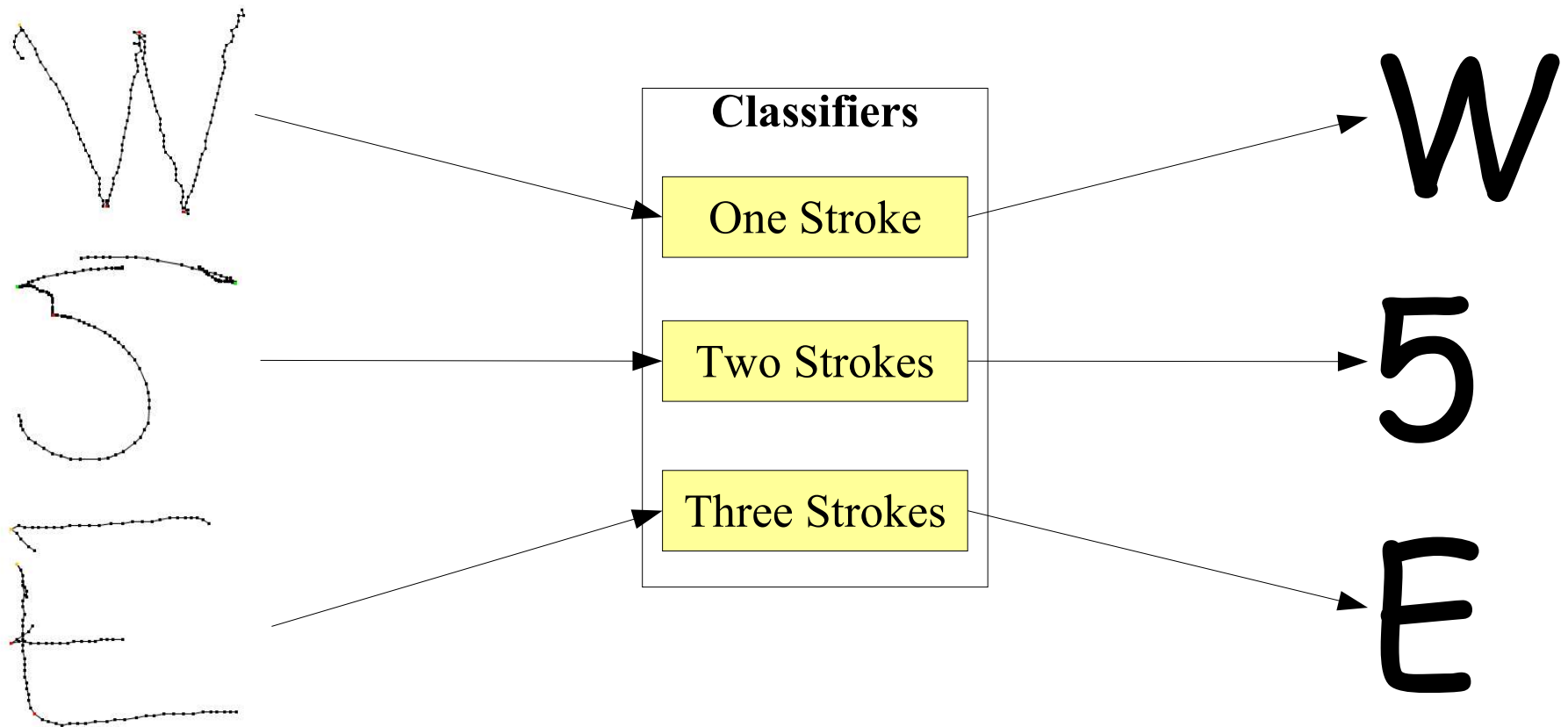
- Coordinates (x_i, y_i)
- Turning angle θ_i
- Change of turning angle $\theta_{i+1} - \theta_i$



Global features:

- Center of gravity $1/n \sum_{i=1}^n p_i$
- Length: L
- Relative length: L/d
- Accumulated angle $1/2 \pi \sum_{i=1}^n \theta_i$

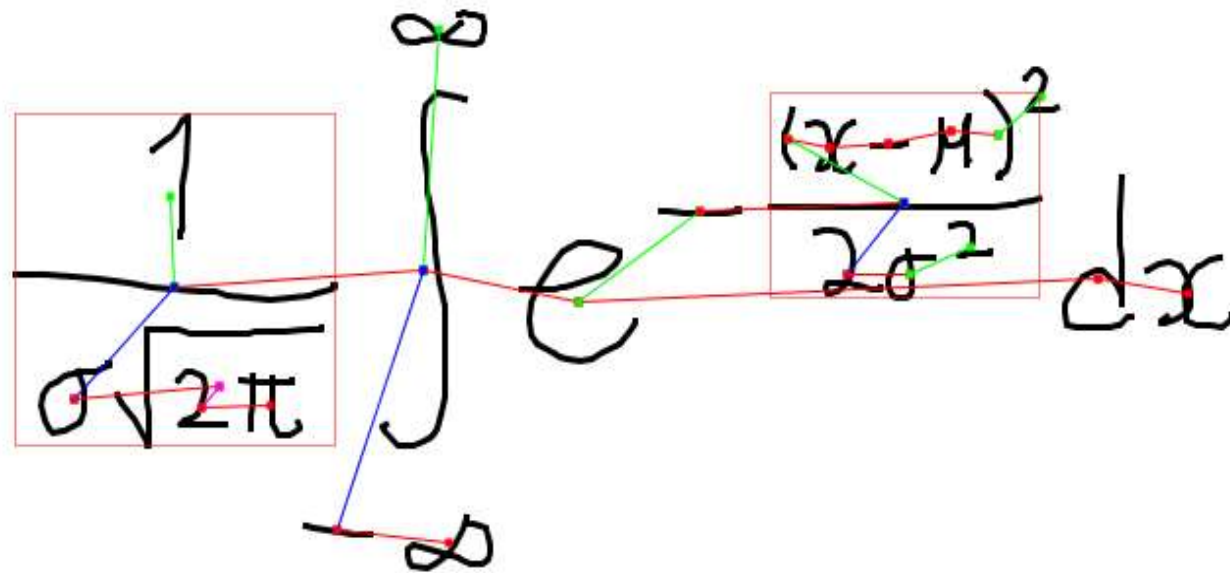
Symbol Recognition: Classification



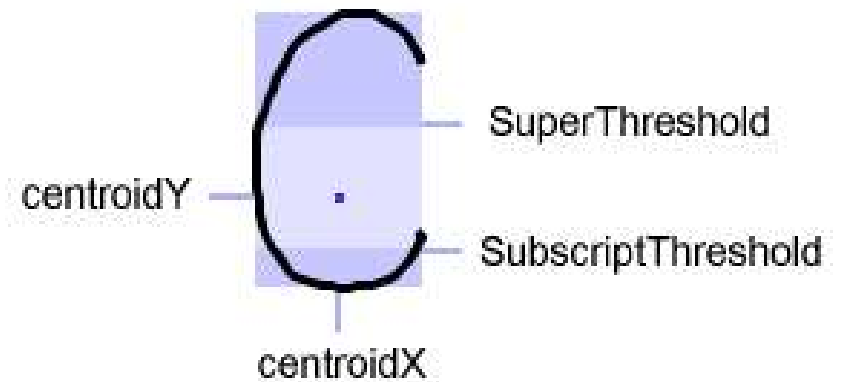
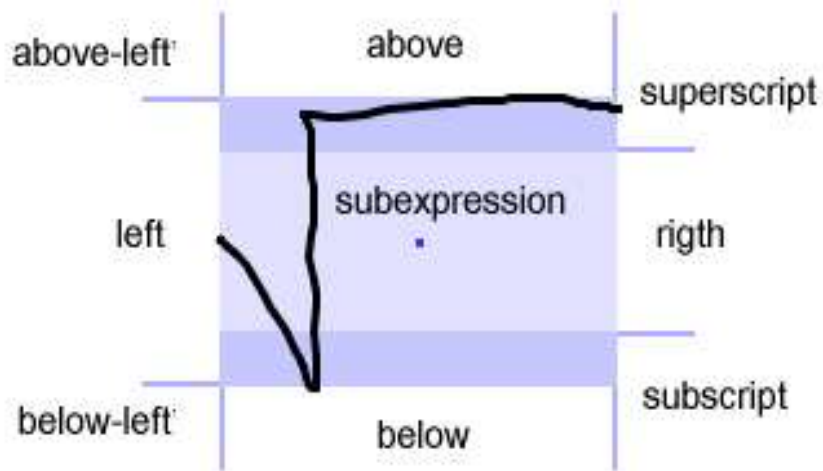
Classification uses support vector machines or artificial neural networks

Structural Analysis: Representation of mathematical Expressions

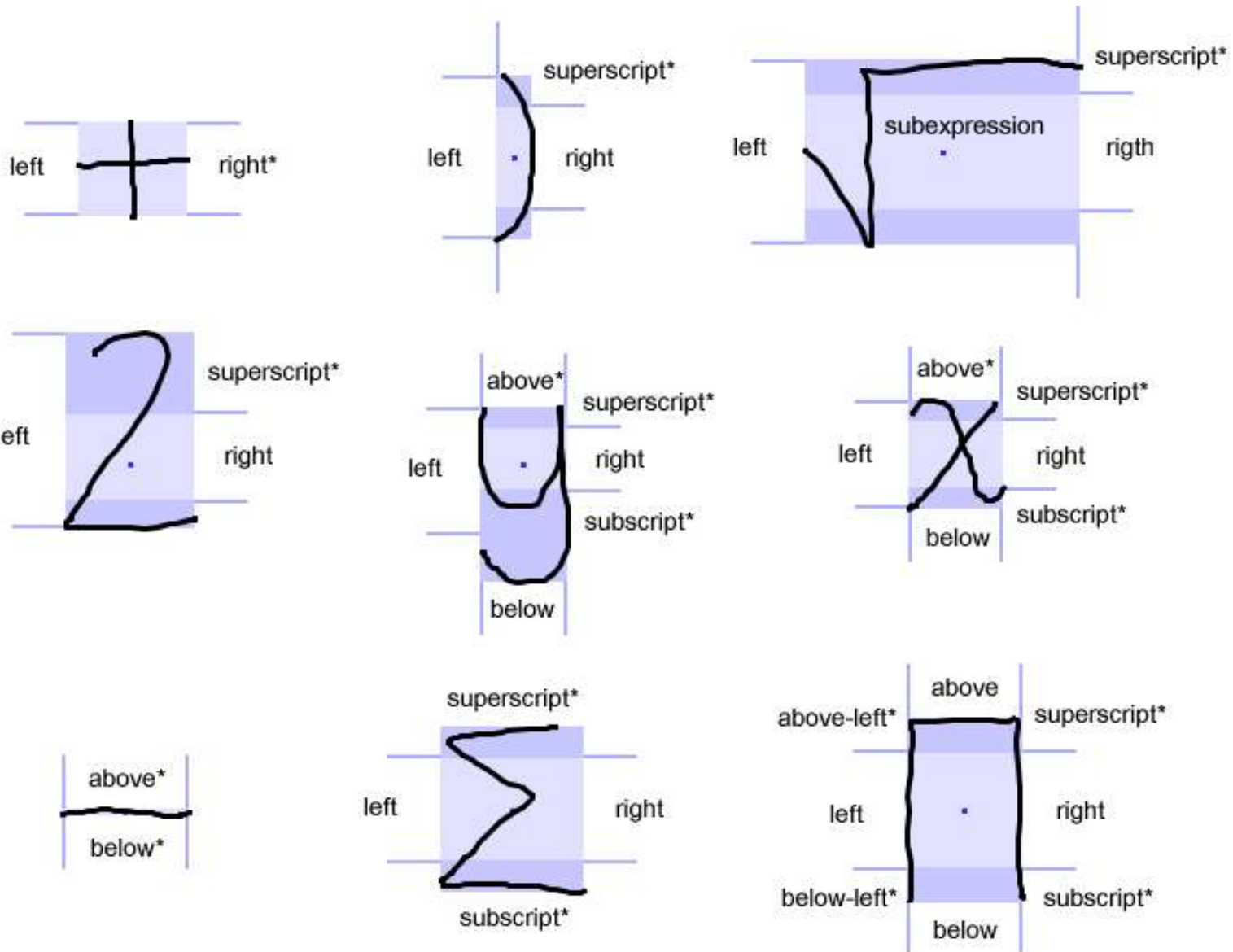
An Expression is a structure of connected baselines



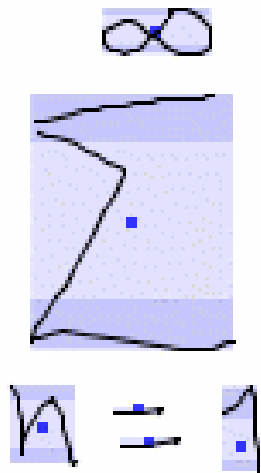
Structural Analysis: Spacial Relations and Attributes



Structural Analysis: Dominance



Structural Analysis: Dominance



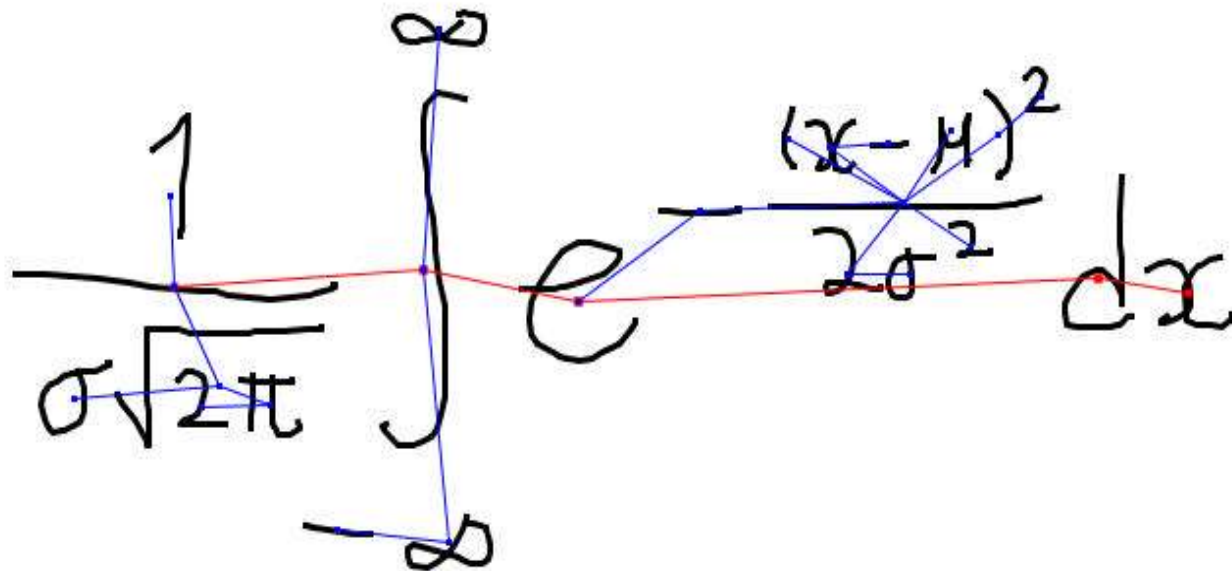
$$\frac{1}{2^n}$$

A circle with a blue dot inside. To its right is the equation $\int x dx$.

$$a + \frac{b}{c}$$

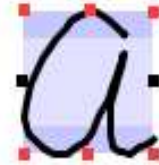
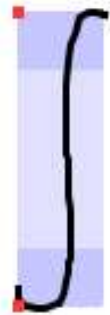
Structural Analysis

- Find the dominant baseline
- Locate clusters of symbols
- Find dominant baselines in clusters



Structural Analysis: Attractor Points

Attractor Points for different classes of symbols:



Structural Analysis: Matrices

- Construct dynamically Attractor Points
- Reduce the horizontal distance

$$\begin{bmatrix} x^3 & 3x^2 & 6x \\ 3x^2 & 6x & 6 \\ 6x^2 & 6 & 0 \end{bmatrix}$$

Structural Analysis: Interpretation

- Typesetting

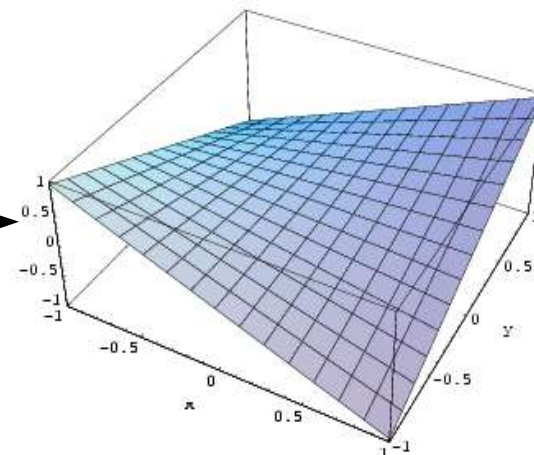
$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \xrightarrow{\text{LaTeX}} \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(\left(x-\mu \right) ^2}{2\sigma ^2}} dx$$

- Computer Algebra Systems

$$\int x^n dx \xrightarrow[\text{s}]{\text{Calculu}} \frac{x^{n+1}}{n+1}$$

$$f(x,y) = x * y$$

Plottin
g



An Editor for on-line handwritten
mathematical Expressions...

Ende