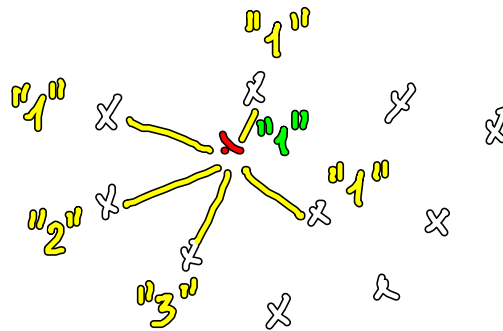
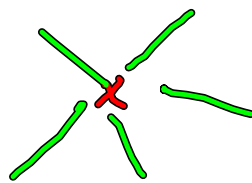


Klassifikatoren

k-NN Klassifiziert



55,000 Ziffern

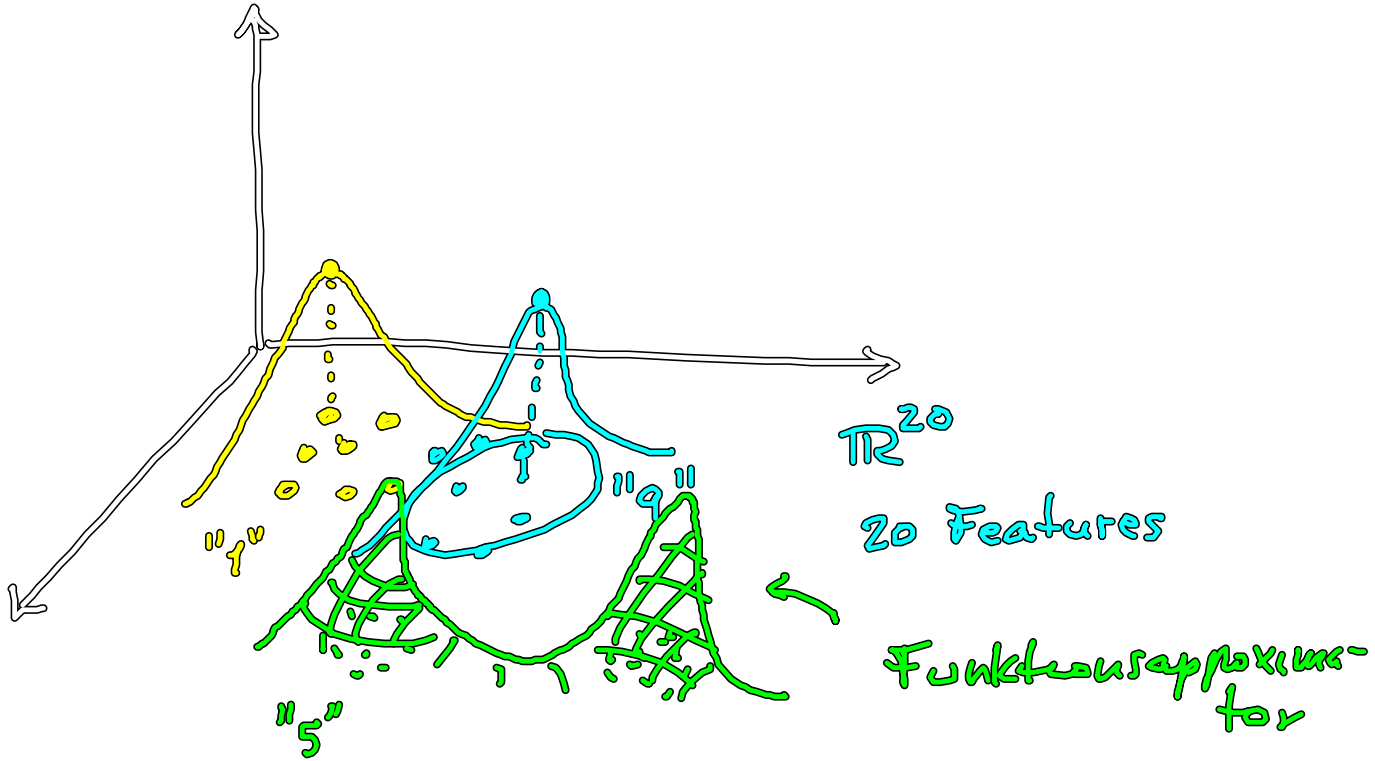


55,000

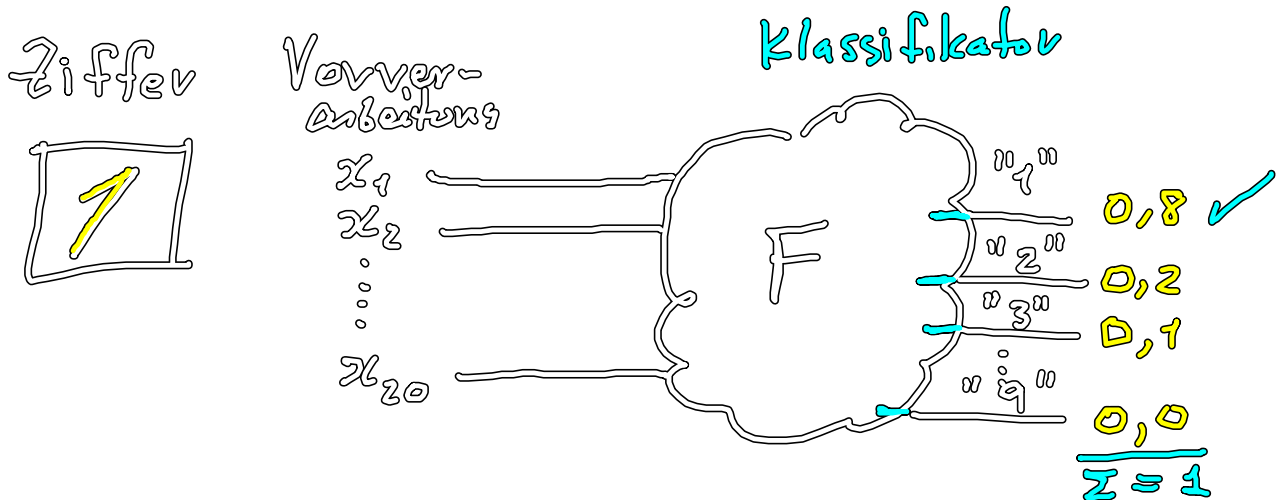
$$\sqrt{(x_1 - y_1)^2 + \dots + (x_n - y_n)^2}$$

$$55,000 \times 20 \approx 10^6$$

100,000 Ziffern



Allgemeiner Klassifikator

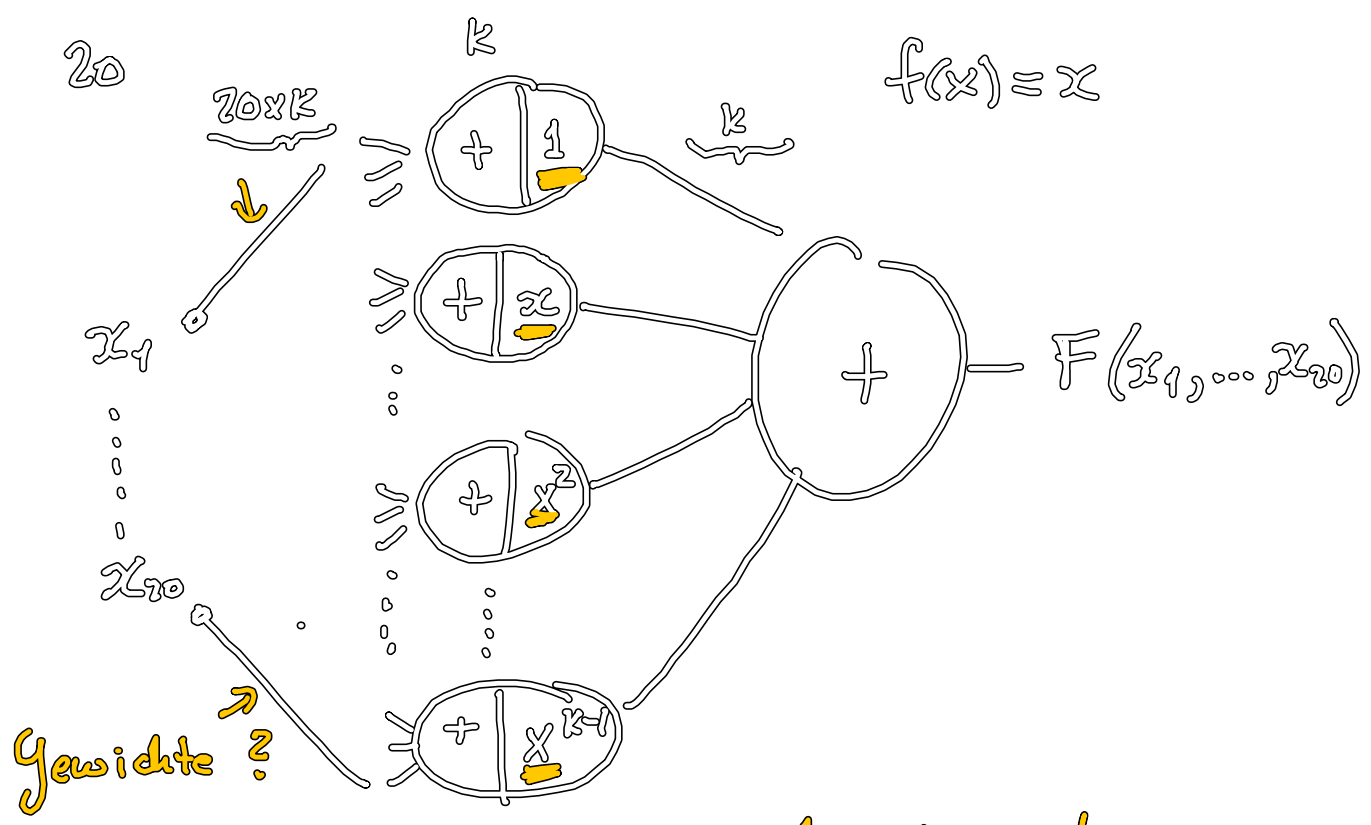
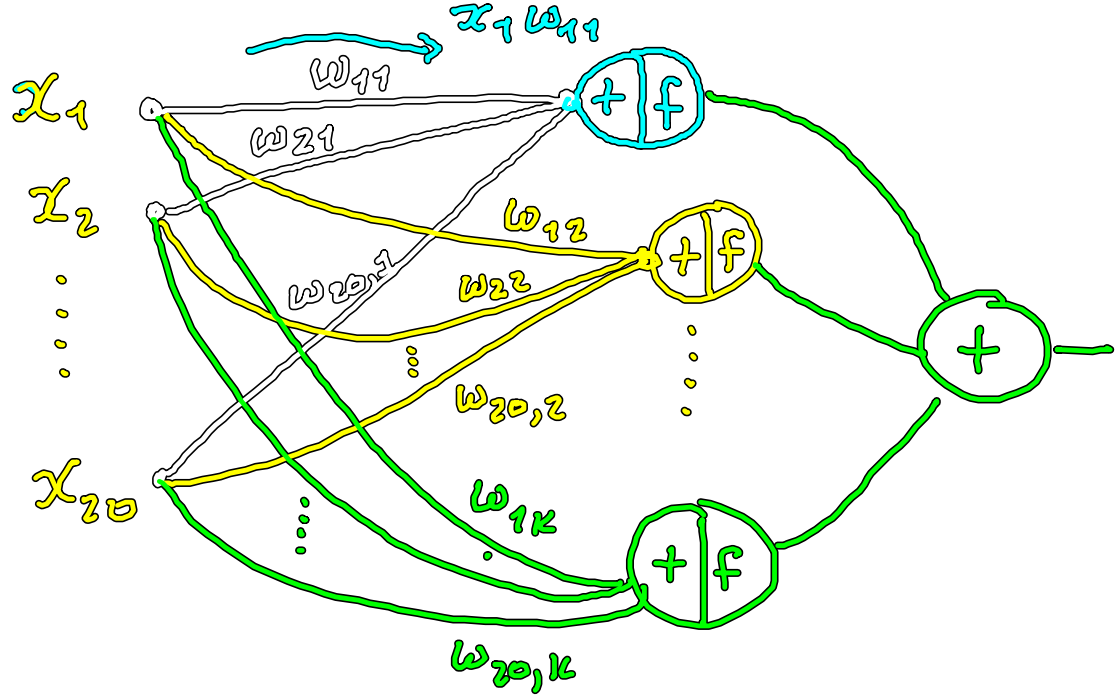


$$F_1(x_1, x_2, \dots, x_{20}) = 0,8$$

$$F_2(x_1, x_2, \dots, x_{20}) = 0,2$$

\vdots

Funktionsnetz = Neuronales Netz

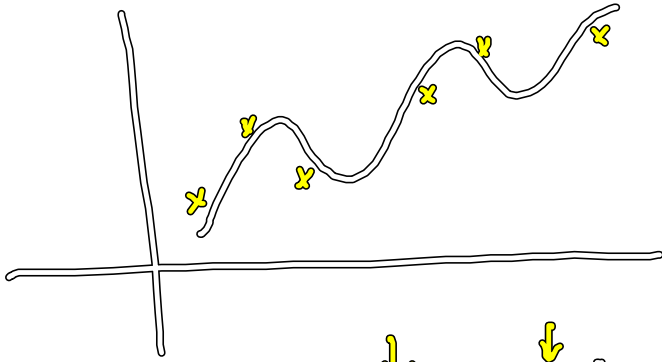


$$F(x_1, \dots, x_{20}) = a_0 + a_1 x_1 + a_2 x_2 \dots$$

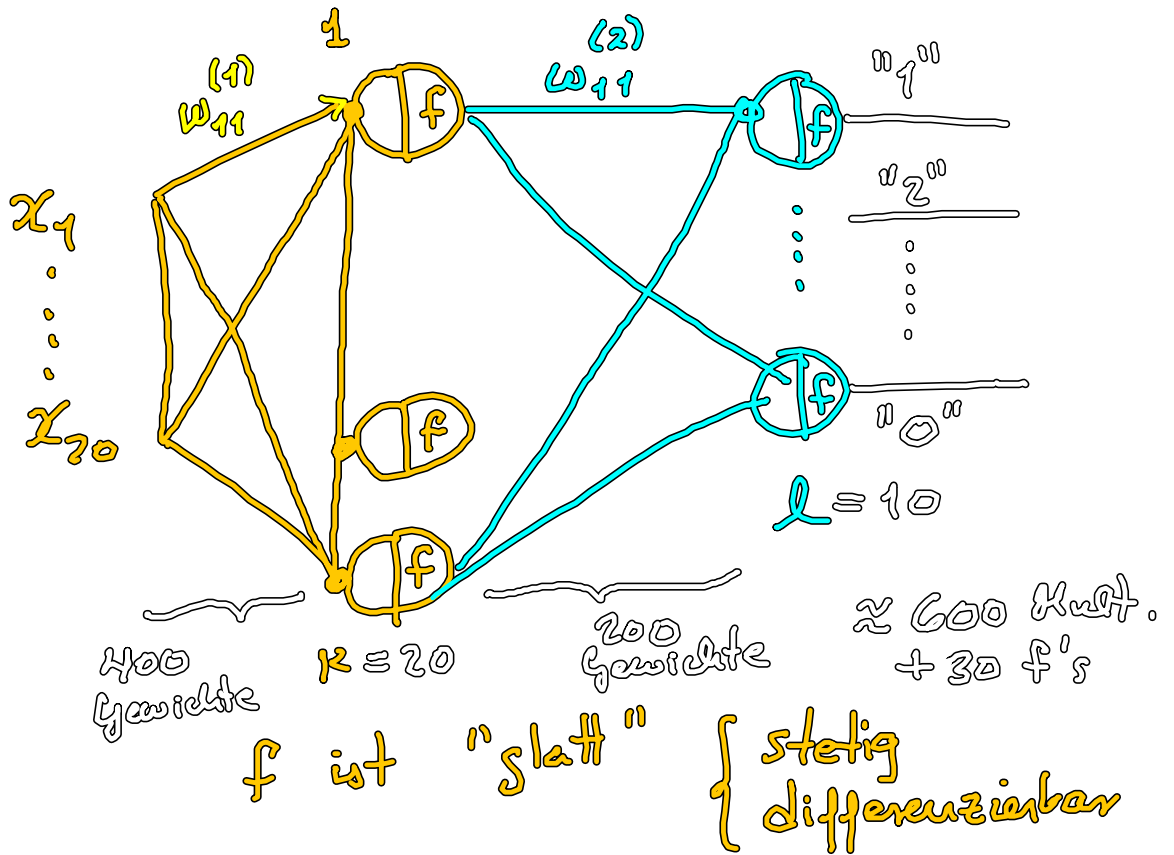
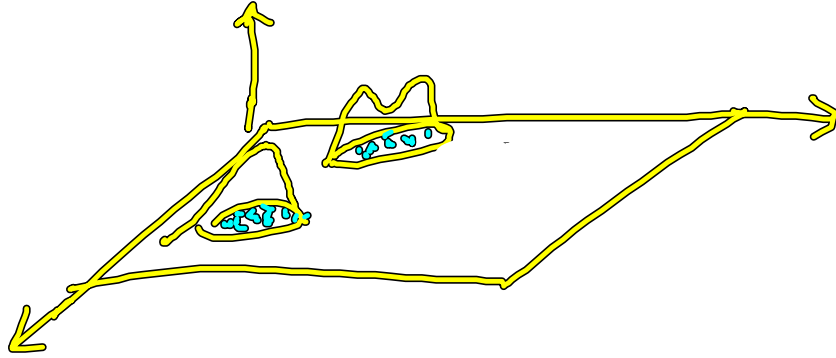
$$\dots + a_{20} x_{20} + a_{21} x_1^2 + \dots$$

$$\dots + a_{40} x_{20}^2 + \dots x_1 x_2 \dots$$

$$\dots x_1^{k-1} + \dots$$



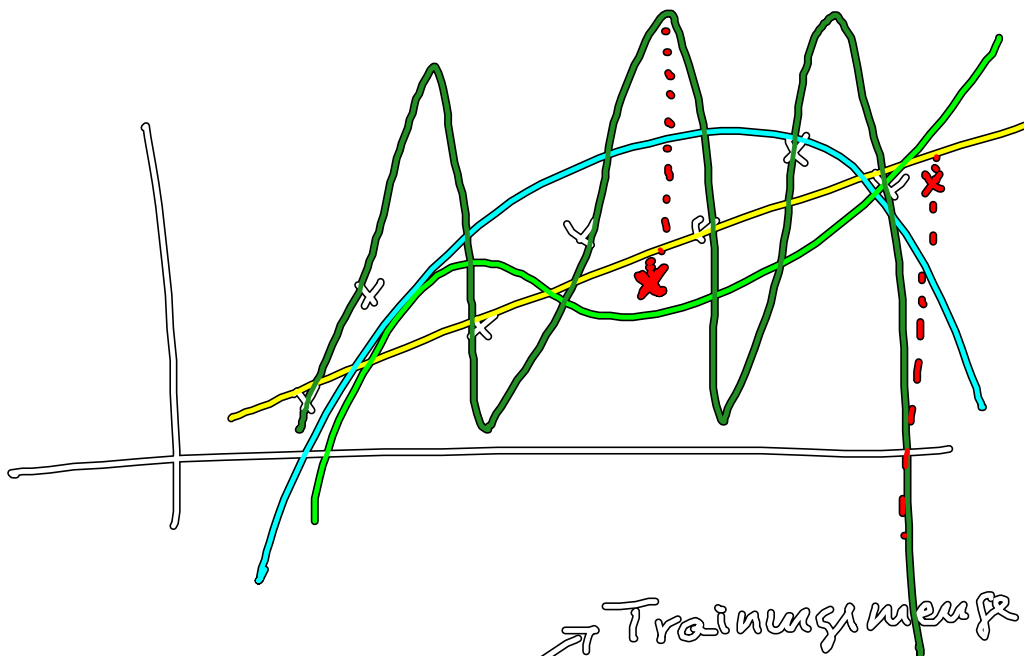
$$1x^2 + 1x^3 + 4x^4 \dots$$



10⁶ 100,000 Ziffer

vs. 600

K-NN vs. NN
genauer vs. schneller
genau



Fehler → Trainingsmenge
 → Recall (neue Eingaben)

Bestes Modell + Einfachstes

Trainingsmenge

100,000 Ziffern



Merkmale



Gewichte



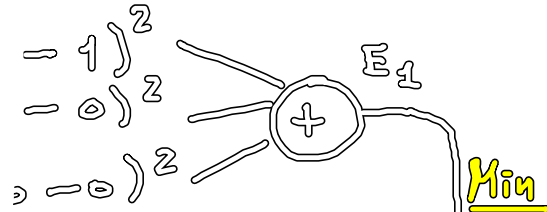
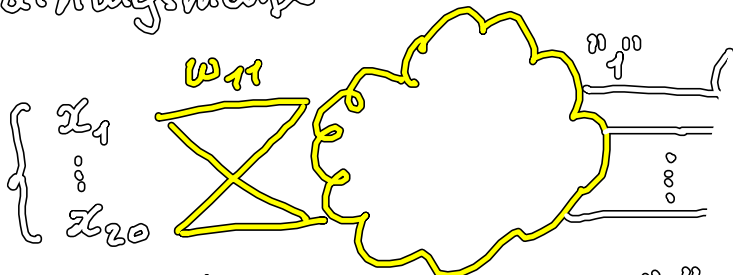
Recall

- zufällige Gewichte
 - testen
- Fehler der Trainingsmenge
- Fehler $> \epsilon$
- Konvergieren
(bis Fehler $< \epsilon$)

Trainingsmenge

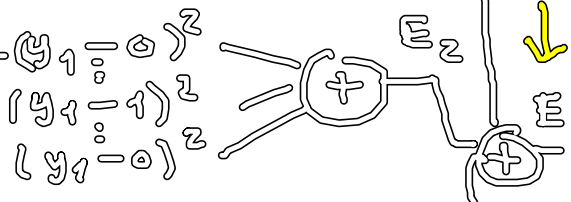
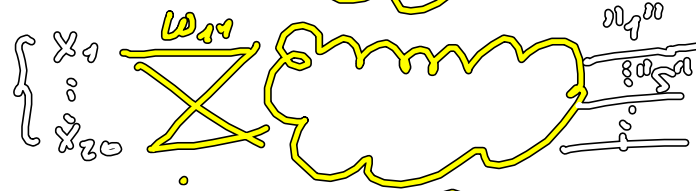
"1"

1.



"5"

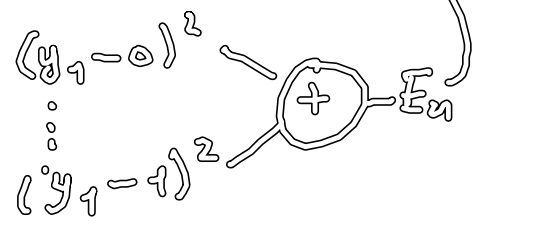
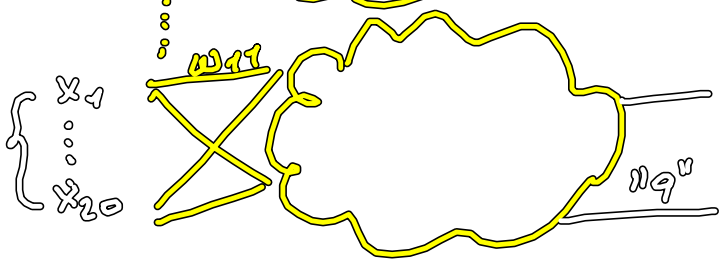
2.



...

"9"

n.

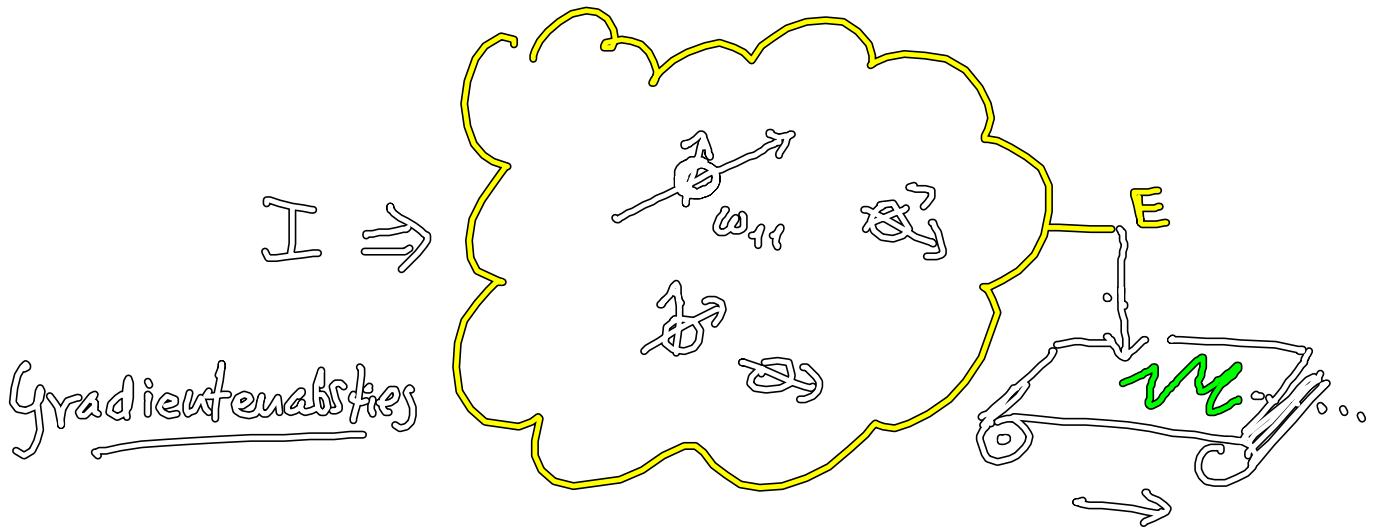
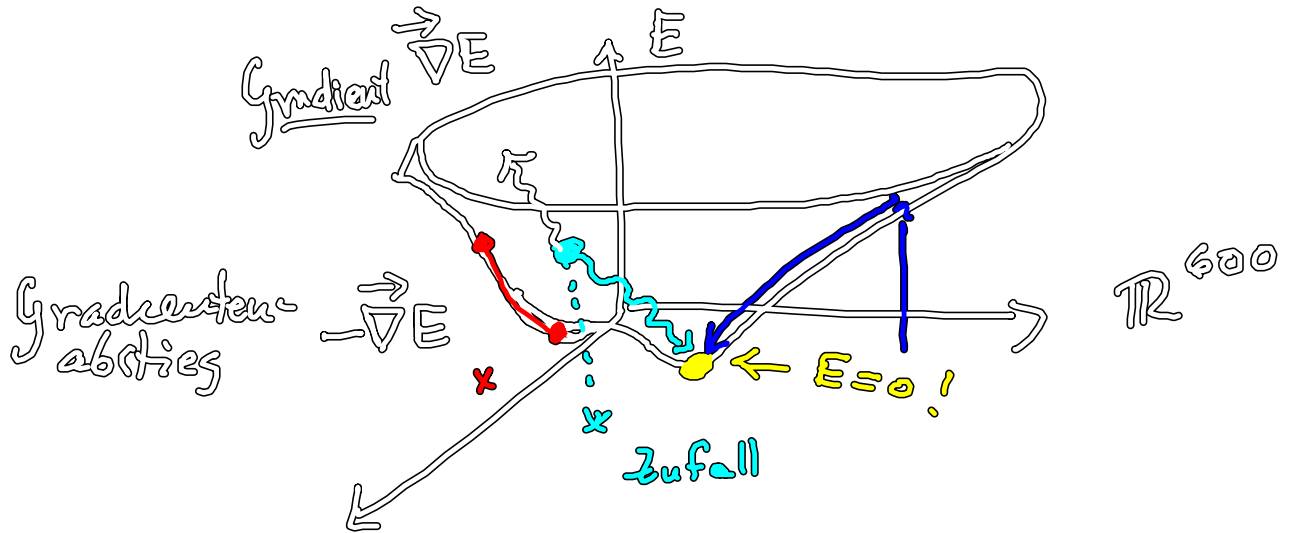
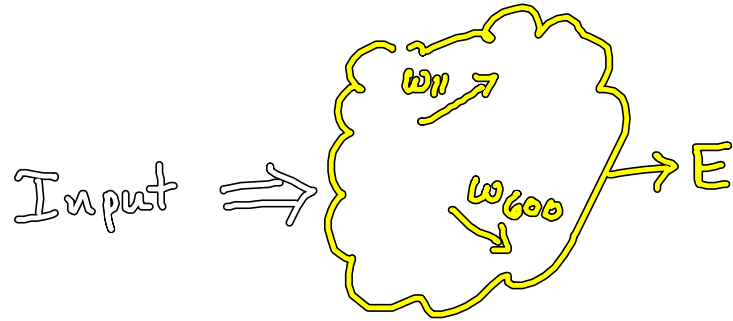


Min



ϵ

Netz



$$\frac{\partial E}{\partial w_{11}}$$

...

$$\Delta w_{11} \uparrow$$

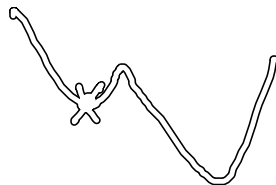
\uparrow

$$\Delta E \uparrow$$

$$\Delta w_{11} \downarrow$$

\downarrow

$$\Delta E \downarrow$$



$$\frac{\partial E}{\partial w_{11}} \dots \frac{dE}{dw_{11}}$$

↓

$$E(w_{11}, w_{12}, \dots, w_{600}) \quad E(w_{11}, \overbrace{w_{12} \dots w_{600}}^{\text{constant}})$$

↑ ↑

Backpropagation

Systematisch

Vorwärtsschritt

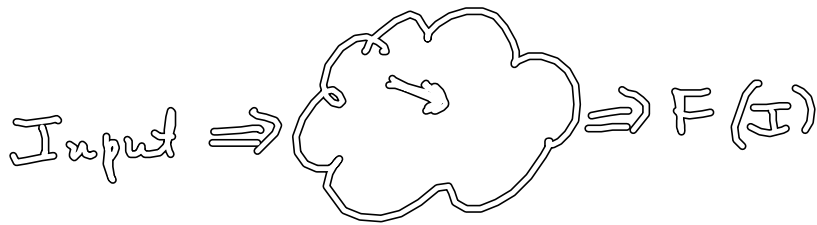
$$x \xrightarrow{\omega} wx$$

Backprop-Schritt

$$\omega \xleftarrow{\omega} 1$$

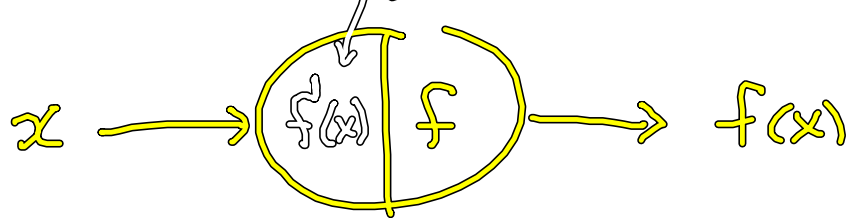
Traversierungswert

$$\frac{d(wx)}{dx} = \omega$$

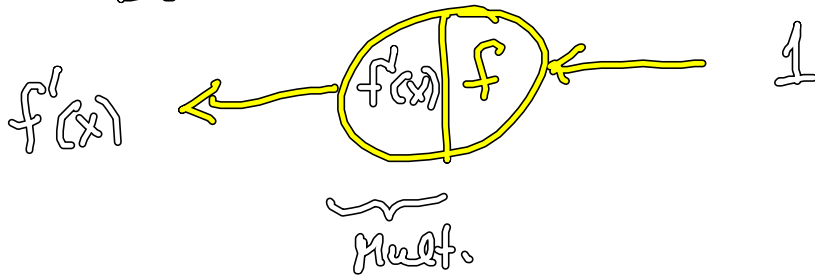


$$\frac{dF}{dI}$$

Vorwärts gespeichert

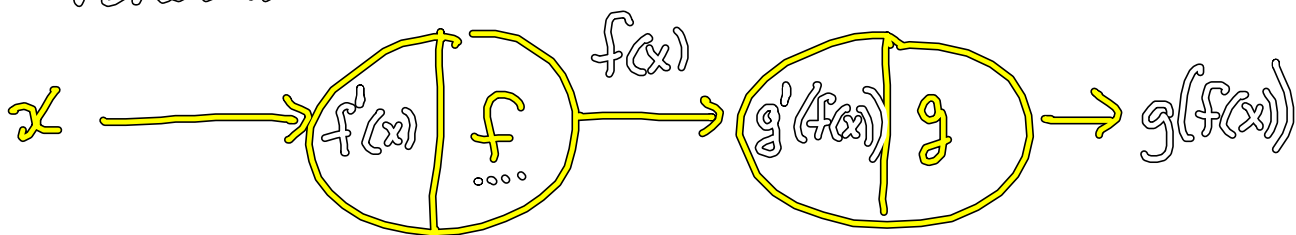


Rückwärts

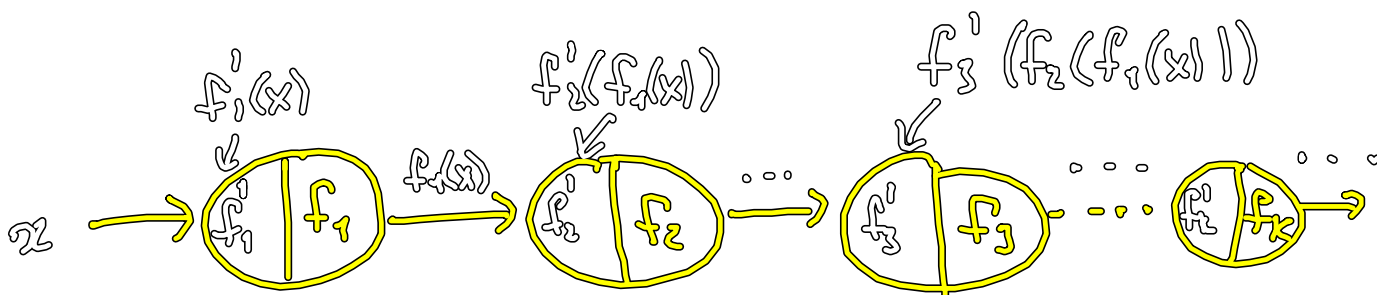
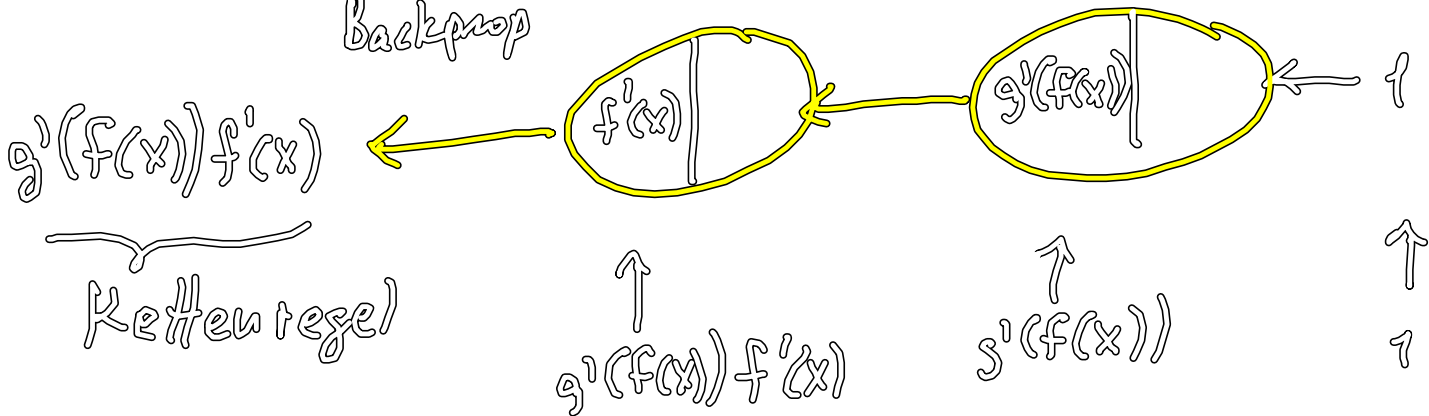


$$\frac{d(f(x))}{dx} = f'(x)$$

Vorwärts

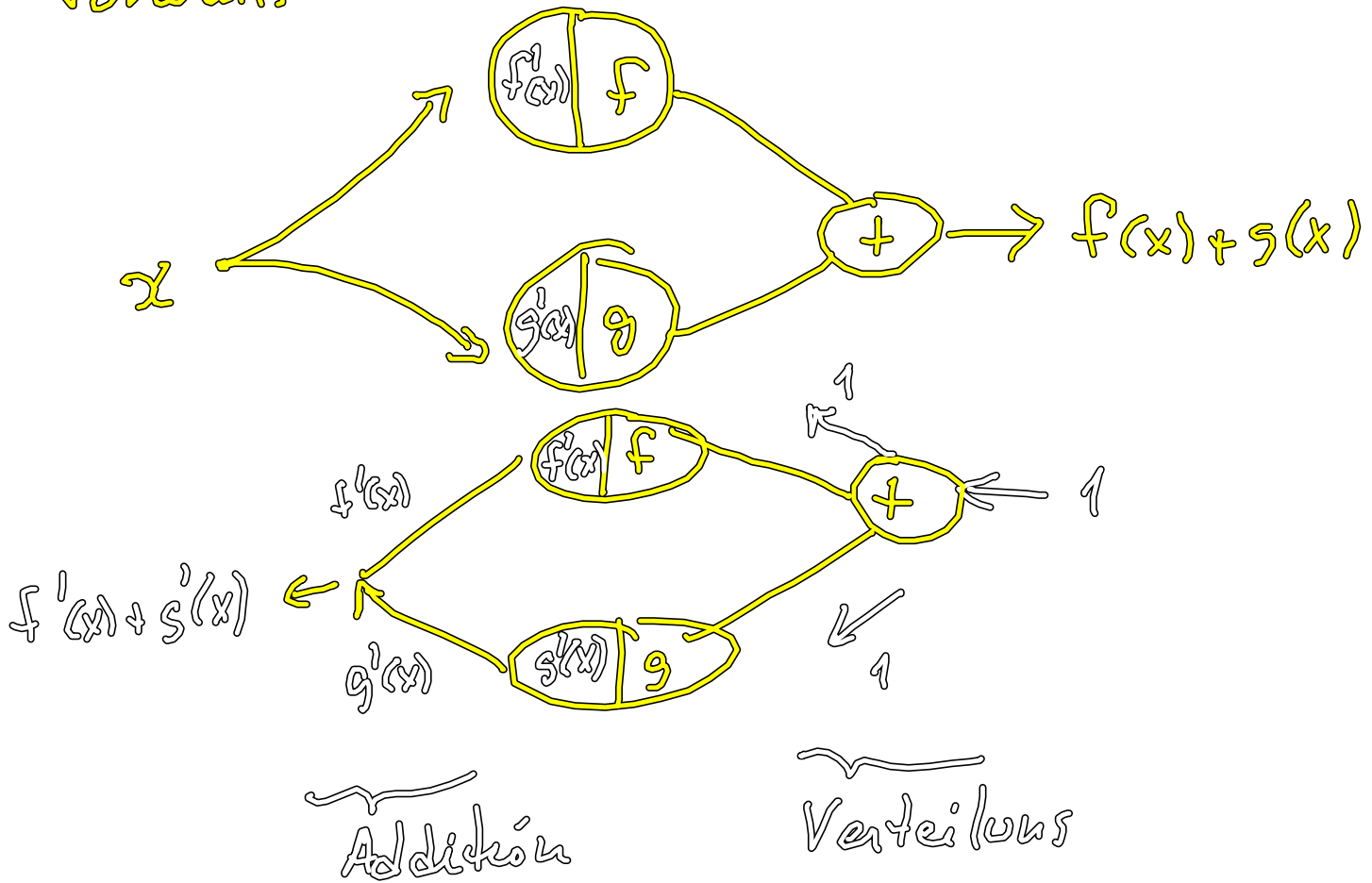


Backprop



per Induktion beweisen...

Vorwärts



$$\frac{d}{dx} (f(x) + g(x)) = f'(x) + g'(x)$$

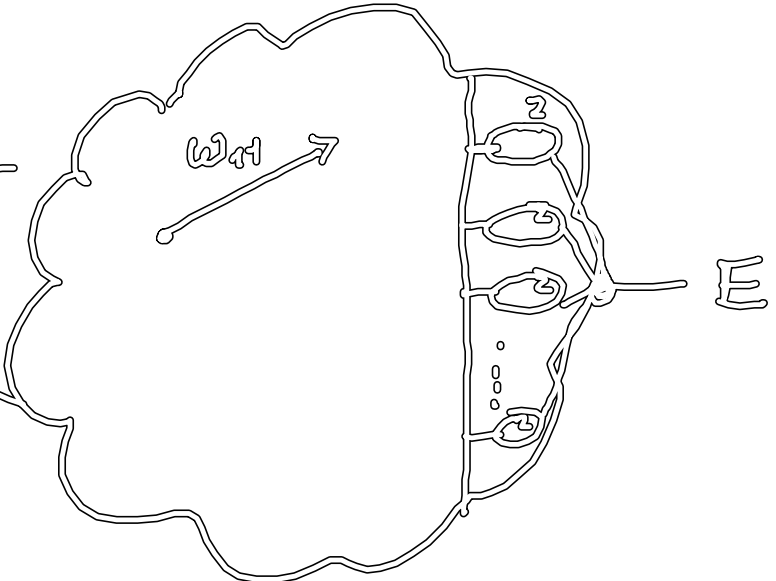
Jetzt an den Rückwärts..

Trainingsmenge

'1'

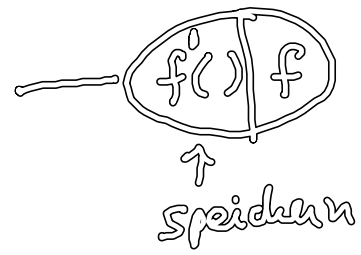
$$\begin{cases} x_1 \\ \vdots \\ x_{20} \end{cases}$$

Konstant

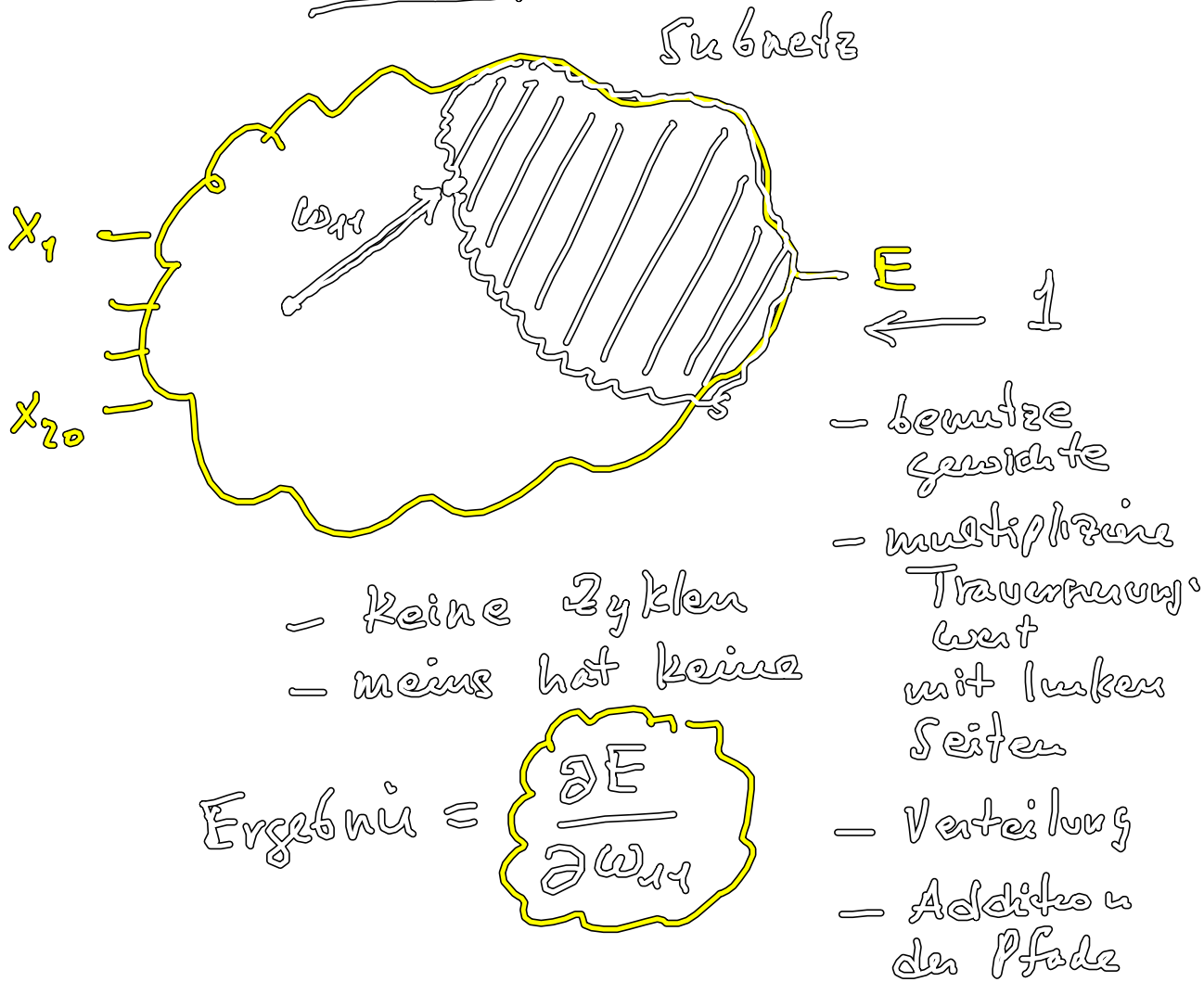


$$\vec{\nabla} E = \left(\frac{\partial E}{\partial w_{11}}, \frac{\partial E}{\partial w_{12}}, \dots, \frac{\partial E}{\partial w_{...}} \right)$$

1. Schritt : Vorwärtslauf



2. Schritt : Backprop.



$$\vec{\nabla} E = \left(\frac{\partial E}{\partial \omega_{11}}, \frac{\partial E}{\partial \omega_{12}}, \frac{\partial E}{\partial \omega_{13}}, \dots, \frac{\partial E}{\partial \omega_{1n}} \right)$$

für alle Gewichte

3. Korrektur :

$$\omega_{11} := \omega_{11} - \delta \frac{\partial E}{\partial \omega_{11}}$$

Konstante (Schrittweite)
↓

$$\omega_{12} := \omega_{12} - \delta \frac{\partial E}{\partial \omega_{12}}$$

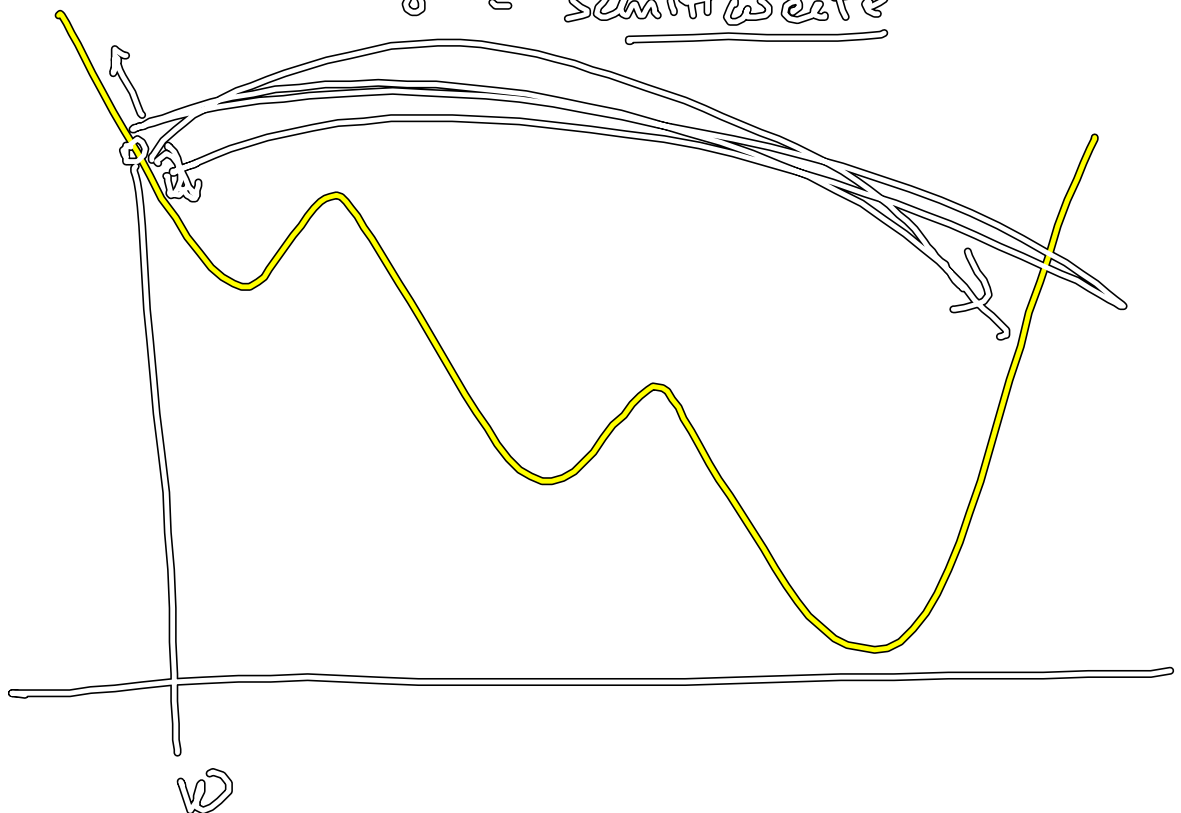
$$\omega_{600} := \omega_{600} - \delta \frac{\partial E}{\partial \omega_{600}}$$

4.- Teste Fehler

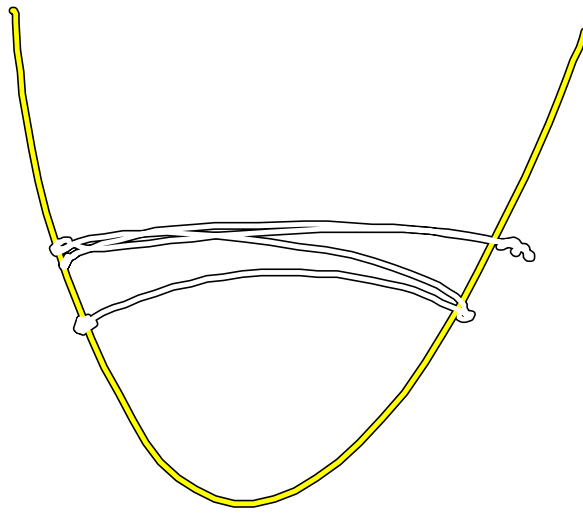
groß \rightarrow weiter bei (1)

Klein genug \rightarrow Stopp.

$\delta =$ Schrittweite



§
...



Zusammenfassung

→ Wir modellieren die Funktion

$$P \begin{cases} P_0(x_1, \dots, x_{20}) \\ P_1(x_1, \dots, x_{20}) \\ P_2(x_1, \dots, x_{20}) \\ \vdots \\ P_9(x_1, \dots, x_{20}) \end{cases}$$

→ Gradientenabstieg für die
Justierung der Gewichte

→ Es gibt keine Garantie für
globales Min.

↳ on-line (G.A. Randomized)
↳ off-line