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Fuzzy Logic

Fuzzy logic is a generalization of classical logic introduced by **Lotfi Zadeh** (1921–) in the mid-1960s. Its purpose is to model those problems in which imprecise data must be used, or in which the rules of inference are formulated in a very general way, making use of diffuse categories. In fuzzy logic, sometimes called *diffuse logic,* there are not just two alternatives but a whole continuum of truth values for logical propositions. A proposition can have the truth value 0.4 and its complement the truth value 0.5. According to the type of negation operator that is used, the two truth values need not necessarily add up to 1.

Fuzzy logic has a weak connection to probability theory. Probabilistic methods that deal with imprecise knowledge are formulated in the Bayesian framework, but fuzzy logic does not need to be justified using a probabilistic approach. The common route is to generalize the findings of multivalued logic in such a way as to preserve part of the algebraic structure of this type of logic.

Multiple-valued logic has a long history. Aristotle raised the question of whether all valid propositions can only be assigned the logical values true or false. The first attempts at formulating a multiple-valued logic were made by logicians such as the Scotsman Hugh MacColl (1837–1909) and the American Charles Sanders Pierce (1839–1914) at the end of the nineteenth and beginning of the twentieth century. However, the first well-known system of multiple valued logic was introduced by the Pole Jan Lukasiewicz (1878–1956) in the 1920s. By defining a third truth value, Lukasiewicz created a system of logic which was later axiomatized by other authors. From 1930 onward, renowned mathematicians such as Kurt Gödel (1906–78), Luitzen Brouwer (1881–1966), and **John von Neumann** (1903–57) continued work on developing an alternative system of logic, which could be used in mathematics or physics. In their investigations they considered the possibility of an infinite number of truth values.

Fuzzy logic can be used for fuzzy control. An expert in a certain field can produce a simple set of control rules for a dynamical system with less effort than when an analytical solution is needed. A classical example, proposed by Zadeh, is developing a control system to park a car. It is straightforward to formulate a set of fuzzy rules for this task, but it is not immediately obvious how to write a standard computer program for the same purpose. Fuzzy logic is now being used in many products of industrial and consumer electronics for which a good enough control system is sufficient and where the question of optimal control does not necessarily arise.

Fuzzy logic starts by defining a fuzzy set theory. The difference between crisp (i.e., classical) and fuzzy sets is established by introducing a *membership function.* For example, in classical logic, a person is either young or old. If the person is young, we say that its membership value regarding the set of young persons is 1, and its membership value regarding the set of old persons is 0. But in fuzzy set theory we can distinguish grades of membership. A person who is 10 years old would receive the same membership values as above, but a person who is, say, 40 years old would belong with a membership value of 0.5 to the young persons and with a membership value of 0.5 to the old persons. A person who is 70 years old would have membership values of 0 and 1, respectively. Obviously, it is not possible to define a definite age that represents the absolute threshold to become old. Aging can be interpreted as a continuous process in which the membership of a person goes slowly from 0 to 1.

Consider now classical logic. In this framework, a proposition is always either true (which we will code using a 1) or false (coded as 0). There is no room for ambiguity. However, in fuzzy logic we accept truth values between 0 and 1. For example, the proposition "this room is cold" would be assigned a 1 or a 0 in classical logic. But in fuzzy logic we would distribute the truth values according to the temperature. A temperature of

10 degrees Celsius (50 degrees Fahrenheit) would produce a truth value of 1 for the statement above. A temperature of 30° Celsius (86° F) would have the truth value 0. But a temperature of 20° C (68° F) would produce, for example, the truth value 0.5. Fuzzy logic therefore tries to model the subjective meaning that persons usually give to certain words. The statement above can be made more complicated by saying "this room is very cold," or "this room is somewhat cold." Fuzzy logic tries to model these *linguistic variables* by assigning them a precise mathematical meaning.

The main complication when using fractionary truth values is how to define the usual logical operations, such as AND, OR, and NOT. However, this can be done just by generalizing these operations in the following way. Let us consider first the conjunction operation, AND. Given two propositions A and B, we can find the truth value of "A AND B" by using a table. The result is only 1 when both statements have the truth value 1 (true); otherwise the result is 0 (false). This can be considered to be the result of computing the minimum of the two truth values because whenever one of the truth values is 0, the result is 0. Using this insight, we can then define the conjunction of two fuzzy truth values A and B as "minimum(A,B)." The fuzzy AND (f-AND) of the truth values 0.7 and 0.5 would thus be minimum(0.7, 0.5) = 0.5.

We define the fuzzy OR (f-OR) in a similar way, as the maximum of the truth values of the two propositions. In the case of negation, we observe that the negation of 1 is 0, and vice versa. That is, the negation of a truth value is 1 minus the truth value. The fuzzy negation (f-NOT) of a truth value A is therefore defined as "1 − A".

Having conjunction, disjunction, and negation, arbitrarily complex propositions can be formed, such as "the room is cold AND the person in the room is old." Although each of the two propositions alone has a fractionary truth value, we can now assign a precise value to the entire sentence (in this case, the minimum of the truth values of every sentence by itself).

Now it is possible to explain how a fuzzy controller works. Assume that we want to build a self-regulating heating unit. We distinguish two types of temperatures, cold and warm. Assume that the membership functions for the temperatures (in parentheses, first for the class "cold," then for the class "warm") are as follows: 10°C (1.0,0.0), 15°C (0.7, 0.3), 20°C (0.5,0.5), 25°C (0.3, 0.7), 30°C (0.0,1.0). Now we formulate two simple control rules: (a) "If the temperature is cold, increase the burning rate," and (b) "If the temperature is warm, decrease the burning rate." When the heater is started, the temperature is measured. Assume that it is 10°C. Since the membership of 10°C to the class of cold temperatures is 1.0, we apply rule (a) with weight 1. Since the membership of 10°C to the class of warm temperatures is 0.0, we also apply rule (b) but with a weight of 0. The net effect is that the burning rate will be increased. If the temperature is 30°C, the opposite occurs: Rule (a) is valid with weight 0.0 and rule (b) is valid with weight 1.0 (i.e., the burning rate is decreased). However, consider the case of a temperature of 20°C. In this case both rules are valid with the same weight 0.5 (i.e., they cancel each other) and the burning rate now remains constant. The control unit brings the room to a comfortable temperature starting from any given temperature.

It is possible to think of many other problems in which we can formulate such fuzzy rules. Balancing a pole vertically in the hand, for example, could be done by formulating the two rules: (a) "If the pole is falling forward, or if it is inclined forward, move the hand forward"; (b) "If the pole is falling backward, or if it is inclined backward, move the hand backward." Notice that the condition for the rule is now an OR combination of two conditions combining the inclination angle with the angular velocity of the pole. Before applying both rules, the fuzzy truth value of the composite condition has to be computed.

These simple examples suffice to give an idea of the type of linguistic rules used in the design of fuzzy controllers. Of course, the same problem could be solved by finding a formula that relates the heating rate with the burning rate, or the angle of inclination and angular velocity of the pole with the velocity of the hand, but this is much more complicated than providing linguistic rules. Also, the controller is easy to understand and modify. Fuzzy controllers are more "transparent" than analytical solutions, which can be optimal but more difficult to develop in the case of real-world problems.

In the 1970s the interest in fuzzy logic and its possible use in expert systems grew rapidly, so that the number of papers published on this topic increased almost exponentially in the ensuing decades. Interest in fuzzy controllers has also augmented dramatically. Some companies already offer microchips with hardwired fuzzy operators and fuzzy inference rules.

Further Reading

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