Semester Report WS05/06 of Taral Guldahl Seierstad

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Topic:	Random graphs and random graph processes
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Field of Research

My field of research is random graphs and random graph processes. The previous semester I have mostly been studying the critical phase in random graphs with a prescribed degree sequence.

An *asymptotic degree sequence* is a sequence of integer valued functions $\mathcal{D} = \{d_0(n), d_1(n), d_2(n), \ldots\}$, such that $d_i(n) = 0$ for $i \geq n$ and $\sum_{i \geq 0} d_i(n) = n$. We let $G_n = G_n(\mathcal{D})$ be a graph chosen uniformly at random from the set of graphs with n vertices, which have $d_i(n)$ vertices of degree i for $0 \leq i < n$. We say that G_n satisfies a property *asymptotically almost surely*, abbreviated a.a.s., if the probability that G_n satisfies the property tends to 1 as n tends to infinity.

It is assumed that the asymptotic degree sequence $\mathcal{D}$ satisfy certain regularity conditions, which are introduced in [1]. In particular it is assumed that the value $\lambda_i(n) := d_i(n)/n$ tends to a constant λ_i^* for every $i \geq 0$ as n tends to infinity. Let $Q(\mathcal{D}) = \sum_i i(i-2)\lambda_i^*$. Molloy and Reed [1] proved that under these conditions, if $Q(\mathcal{D}) < 0$, then a.a.s. all the components in G_n are *small*, in particular all components have $O(\omega(n)^2 \log n)$ vertices, where $\omega(n)$ is the maximal degree of G_n; while if $Q(\mathcal{D}) > 0$, then G_n a.a.s. contains a unique giant component with $\Theta(n)$ vertices, while all other components have $O(\log n)$ vertices.

The previous semester I have, together with Dr. Mihyun Kang, considered the case when $Q(\mathcal{D}) = 0$, which is not covered in [1]. This is called the critical phase. We let

$$Q_n(x) = \sum_{i \geq 1} i(i-2)\lambda_i(n)x^i,$$

and we let τ_n be such that $Q_n(\tau_n) = 0$. The case that $Q(\mathcal{D}) = 0$ corresponds to the case that $\lim_{n\to\infty} \tau_n = 1$. We studied in what way the structure of G_n depends on how fast, and from which direction, τ_n converges to 1. In particular, we proved the following. Let $\delta_n = 1 - \tau_n$.

(1) If $\delta_n n^{1/3} \to -\infty$, and $|\delta_n n^{1/3}| \gg \log^{1/4} n$, then a.a.s. all components in G_n have order $o(n^{2/3})$.

(2) There is a constant c such that if $\delta_n n^{1/3} \geq c \log n$, then a.a.s. G_n has a single component of order $\gg n^{2/3}$, while all other components have order $o(n^{2/3})$.

Activities

- Attended weekly lectures and colloquia of the CGC.
- Attended weekly seminar of the research group *Algorithmen und Komplexität* at the HU Berlin. (21 October 2005 I held a talk with the title *Recursive trees.*)
- Attended the CGC Annual Workshop 2005 in Hiddensee from 25 until 28 September 2005. (I held a talk with the title *The minimum degree graph process.*)
- Attended the 12th International Conference on Random Structures and Algorithms in Poznań from 1 until 5 August 2005. (4 August 2005 I held a talk with the title *The phase transition in the minimum degree random graph process.*)
- Attended the European Conference on Combinatorics, Graph Theory and Applications (EuroComb 2005) from 5 until 9 September 2005.
- Visited the Adam Mickiewicz University in Poznań from 28 November until 2 December 2005. 29 November I held a talk in the weekly seminar at the department for Discrete Mathematics, with the title *The critical phase for random graphs with given degree sequence.*
- In the beginning of January 2006 I started my long stay at the Adam Mickiewicz University in Poznan.

Preview

Until the end of May I will stay at the Adam Mickiewicz University in Poznań, and work with Professor Łuczak and Professor Ruciński. I will continue to study random graph processes. Two problems I will concentrate on are the critical phase in random digraphs, and the subgraph containment problem in the trianglefree random graph process.

References

[1] Michael Molloy and Bruce Reed. A critical point for random graphs with a given degree sequence. In *Proceedings of the Sixth International Seminar on Random Graphs and Probabilistic Methods in Combinatorics and Computer Science, "Random Graphs '93" (Poznań, 1993)*, volume 6, pages 161–179, 1995.