

Semester Report WS05/06 of Jakob Jonsson

Name: Jakob Jonsson
Supervisor: Günter Ziegler
Field of Research: Topological and enumerative combinatorics
Topic: Independence complexes of grid graphs
Postdoc: at the program from July to December 2005

Field of Research

My focus has been on enumerative and topological properties of specific finite simplicial complexes. The complexes were typically defined in terms of combinatorial objects such as graphs.

Results

In my main research project, I studied simplicial complexes of independent sets in square grid graphs with periodic boundary conditions. The latter means that we identify two vertices in the infinite unit square grid whenever they are on distance an integer linear combination of the two vectors $(m, 0)$ and $(0, n)$, where m and n are fixed integers. In a recent preprint [5], I related the Euler characteristic of such complexes to certain periodic tilings of the plane into rhombi and squares, thereby settling two conjectures from statistical mechanics due to Paul Fendley, Kareljan Schoutens, and Hendrik van Eerten [1]. In another preprint [6], I deduced some partial information about the homology of the complexes under consideration. Just recently [7], I examined a “diagonal” variant of this grid in which the two underlying “identification vectors” are $(-m, m)$ and (n, n) . Intriguingly, the asymptotic behavior of the Euler characteristic is completely different in this case compared to the previous case.

In joint work with Volkmar Welker, I examined Pfaffian ideals in certain polynomial rings. Building on work by Herzog and Trung [3] and Ghorpade and Krattenthaler [2], we managed to define a monomial order such that the underlying initial ideal is the Stanley-Reisner ideal of a sphere joined with a simplex. This simplicial complex appeared in an earlier paper [4], and we used our construction to extend some results from that paper. Welker and I also have an ongoing project in which we examine certain boolean complexes defined in terms of injective words.

Activities

Talks

- *Simplicial Complexes of Graphs with a Bounded Covering Number*, Seminar of the Discrete Geometry group at TU Berlin, July.
- *Grid Graphs with Periodic Boundary Conditions and Rhombus Tilings of the Plane*, 5th Workshop on Combinatorics, Geometry, and Computation, Hiddensee, September 27.
- *Hard Squares with Negative Activity and Rhombus Tilings of the Plane*, Colloquium of the European Graduate Program "Combinatorics, Geometry, and Computation", TU Berlin, November 28.

Other activities

- *5th Workshop on Combinatorics, Geometry, and Computation*, Hiddensee, September 25–28.
- *Marburg University*, Mathematical discussions with Volkmar Welker, October 9–14.
- Attended CGC Monday Lectures and Colloquia, October 24–December 12.
- Attended seminars of the Discrete Geometry group at TU Berlin.

Preview

I will spend the first months of 2006 at *Institut Mittag-Leffler* in Stockholm; this is within the 2006 Algebraic Topology program. In mid-March, I will return to Günter Ziegler's group at TU Berlin as a post-doc (though not within the CGC program).

References

- [1] P. Fendley, K. Schoutens, H. van Eerten, Hard squares with negative activity. *J. Phys. A: Math. Gen.* **38** (2005), 315–322.

- [2] S. R. Ghorpade and C. Krattenthaler, The Hilbert series of Pfaffian rings, *Algebra and Algebraic Geometry with Applications*, C. Bajaj, C. Christensen, A. Sathaye and G. Sundaram, eds., Springer-Verlag, New York, 2004, pp. 337-356.
- [3] J. Herzog and N. V. Trung, Gröbner bases and multiplicity of determinantal and Pfaffian ideals, *Adv. Math.* **96** (1992), 1–37.
- [4] J. Jonsson, Generalized triangulations and diagonal-free subsets of stack polyominoes, *J. Combin. Theory, Ser. A* **112** (2005), 117–142.
Preprint available from www.math.kth.se/~jakobj/combin.html .
- [5] J. Jonsson, Hard squares with negative activity and rhombus tilings of the plane, Preprint, 2005.
Available from www.math.kth.se/~jakobj/combin.html .
- [6] J. Jonsson, Certain homology cycles of the independence complex of grid graphs, Preprint, 2005.
Available from www.math.kth.se/~jakobj/combin.html .
- [7] J. Jonsson, Hard squares on grids with diagonal boundary conditions, Preprint, 2005.
- [8] J. Jonsson and V. Welker, A spherical initial ideal for Pfaffians, Preprint, 2005, math.CO/0601335.