

Semester Report WS 05/06 of Stephan Hell

Name:	Stephan Hell
Supervisor:	Prof. Dr. Günter M. Ziegler
Field of Research:	Discrete Geometry
Topic:	Topological Methods in Combinatorics and Geometry
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Field of research and Results

This is my last semester in the program, so that I spent – and will spend – most of my time writing my thesis. I plan to hand it in end of April.

In October and November, I pursued my work on topological fractional Helly theorems which goes back to the summer semester '05 and my preprint [3]. I worked on problems on homological VC-dimension which were proposed by Gil Kalai in [5]. In doing so, I understood several details which were mysterious to me before: 1) Proof of the *Upper bound theorem for d -Leray complexes* using Alexander duality and the characterization of h -vectors of Cohen-Macaulay complexes. 2) Proof of the *Upper bound theorem for d -Leray complexes* using algebraic shifting. 3) Proof that the two ways of introducing Leray complexes coincide using a lemma due to Bayer, Charalambous, and Popescu [1]. 4) Partial answers towards Conjecture 6 of [5]. All of this will be part of my thesis.

In December, I looked again into my results on the number of Tverberg partitions from a computer project started in winter semester '03/'04. In fact, I rewrote parts of it. Moreover, the random maps can now be visualized in the affine case using xfig – a drawing tool which is available for most unix platforms. At the same time, I had discussions with Rade Živaljević on the colored version of the Tverberg theorem. Coincidentally, I looked into [2] on the number of colorful simplices containing the origin. This combination led me to new ideas which will be written up – together with my results obtained in [4] – in my thesis: a parity result for Tverberg partitions, a new lower bound for the number of Tverberg partitions without using topological methods, a spherical Tverberg theorem, a lower bound for the number of colored Tverberg partitions using topological methods. In the remaining time, I will try to get as close as possible to the topological Tverberg in dimension 2, hoping that this leads either to a proof or a counterexample of Sierksma's conjecture for continuous maps.

Acknowledgment

Since this is my last full semester in the program, and this is thus my last semester report, I would like to thank all people from the program *Computation, geometry, and computation*. Special thanks to our coordinator Andrea Hoffkamp, and the speaker of the program Helmut Alt. I am very glad to be a member of this graduate school and of our group *Diskrete Geometrie*. I would also like to thank my advisor Günter M. Ziegler for his great support and encouragement.

Activities

- Attended Lectures and Colloquia of the CGC
- Talk at *Noon seminar* of our group Discrete Geometry, TU Berlin
- Attended lecture *Ausgewählte Kapitel der kombinatorischen Geometrie* of Thorsten Theobald, TU Berlin

Preprint

I plan to visit Rade Živaljević at the Serbian Academy of Sciences, Belgrade, in April. He was a visitor of Günter M. Ziegler's group last December, and he is an expert in topological combinatorics. During his stay, we have started discussing Tverberg-type problems.

References

- [1] D. BAYER, H. CHARALAMBOUS, AND S. POPESCU, *Extremal Betti numbers and applications to monomial ideals*, J. Algebra **221** (1999), pp. 497–512.
- [2] A. DEZA, S. HUANG, T. STEPHAN, AND T. TERLAKY, *Colourful simplicial depth*, Preprint math.CO/0506003, 18 pages. To appear in Discrete and Computational Geometry.
- [3] S. HELL, *On a topological fractional Helly theorem*, 2005. Preprint arXiv.math.CO/0506399.

- [4] S. HELL, *On the number of Tverberg partitions in the prime power case*, Preprint math.CO/0404406, 7 pages. To appear in Europ. J. of Combinatorics.
- [5] G. KALAI, *Combinatorial expectations from commutative algebra*, Tech. Report 32, Math. Inst. Oberwolfach, July 2004. Workshop "Combinatorial commutative algebra" organis. by I. Peeva and V. Welker.