Semester Report WS05/06, Cornelia Dangelmayr

Name:	Cornelia Dangelmayr
Supervisor:	Prof. Stefan Felsner
Field of Research:	Graph Theory
Topic:	Intersection Graphs and Subclasses
PhD Student	associated member at the program since May 2004

Field of Research

I'm investigating graphs which can be represented as intersection graphs of a family of pseudosegments in the plane. Graphs admitting such a representation are called IPS graphs. The class of IPS graphs and the subclass of intersection graphs of straight line segments IS have already received some attention. It's known that:

- triangle-free planar graphs are IS [4]
- three colorable planar graphs are IS [5]
- series-parallel graphs are IS [2]

It has been conjectured that indeed every planar graph is IS. So far, however, nobody has been able to proof the weaker statement, that every planar graph is IPS. Permutation graphs are IS by definition. Likewise it is easy to show, that interval graphs are IPS, they even are IS. Since chordal graphs are a natural superclass of interval graphs, I started investigating whether chordal graphs are IPS.

Results

In regard of chordal graphs I achieved a negative and a positive result. I proof that there exist graphs that are chordal but neither intersection graphs of straight line nor of pseudosegments. Such a familiy form the graphs K_n^3 with $n \geq 39$, obtained by attaching a vertex for every triple $\{i, j, k\} \subset [n]$ to its defining vertices i, j, k of K_n.

These graphs can't be represented as intersection graphs of pseudosegments in the plane as it is not possible to add $\binom{n}{3}$ segments to an arbitrary arrangement of n pairwise intersecting pseudosegments such that they intersect different triples of them for $n \geq 39$. In such an arrangement every pseudosegment corresponding to vertices of $K_n \subset K_n^3$ is cut into n pieces and every triple segment corresponding to the triple vertices of K_n^3 intersects three such pieces. These pieces can be taken as set of vertices V_A and the set of triple segments as set of paths of length two. Then the set of edges of those paths induces a planar multigraph on V_A. Applying the restriction that each pair of triple segments has to be disjoint, one obtains a quadratic upper bound in n for the number of different triple segments that can be added to an arbitrary arrangement of n pairwise intersecting pseudosegments.

On the other hand I amplified the common subclass of chordal graphs and intersection graphs of pseudosegments. I considered the characterization of chordal graphs as vertex intersection graphs of subtrees of a tree $\mathcal{V}T\mathcal{T}$ where the vertices of a graph G correspond to subtrees $T' \subseteq T$ of $\mathcal{T}' \subseteq \mathcal{T}$ such that $vw \in E(G)$ if and only if $T_v \cap T_w \neq \emptyset$ with $T_v, T_w \in T'$. According to this definition interval graphs are vertex intersection graphs of subpaths of a path $\mathcal{V}P\mathcal{P}$. I show that the respective superclass of vertex intersection graphs of subpaths of a tree $\mathcal{V}P\mathcal{T}$ belongs to the class of intersection graphs of pseudosegments. The proof is inductive thus supplying a method to obtain such a representation.

Preview

To investigate further subclasses of chordal graphs and intersection graphs of pseudosegments in respect of this characterisation it may be worth to restrict the number of leaves of $\mathcal{T}$.
The subclasses obtained bounding $\Delta(\mathcal{T}) \leq k$ or $diam(\mathcal{T}) \leq h, h \in \mathbb{N}$ contain K_n^3 for $n \in \mathbb{N}$ with $k \geq 3$ and $h \geq 2$.

Another interesting class emerges observing that interval graphs are co-comparability graphs of orders of interval dimension 1. I will investigate the case of orders of interval dimension 2 and some special cases of it.

Activities

- I attended the Monday Lectures and Colloquia of the CGC and gave a talk.

- I participated in the summer school on „Geometric Combinatorics “in Vienna, July 18th to 29th 2005.

- I attended the European Conference on Combinatorics, Graph Theory and Applications in Berlin, September 5th to 9th 2005
- I participated in the CGC workshop on Hiddensee, September 25th to 28th 2005.
- I took part in the seminar „Partielle Ordnungen "of Prof. Felsner, January 27th to 29th 2006.
- I took part in the weekly noon seminar of the workgroup „Diskrete Mathematik "at the TU.

Literatur

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