

# Semester Report WS05/06 of Hubie Chen

Name: Hubie Chen  
Supervisor(s): Martin Grohe  
Field of Research: Constraint Satisfaction  
Topic: Tractability in Constraint Satisfaction  
Postdoc: at the program from September 2005 to November 2005

## Field of Research

The *constraint satisfaction problem* (CSP) is widely acknowledged as a convenient framework for modelling search problems. Instances of the CSP arise in a variety of domains, including artificial intelligence, database theory, algebra, propositional logic, and graph theory. An instance of the CSP consists of a set of constraints on a set of variables; the question is to determine if there is an assignment to the variables satisfying all of the constraints. Alternatively, the CSP can be cast as the fundamental algebraic problem of deciding, given two relational structures  $\mathbf{A}$  and  $\mathbf{B}$ , whether or not there is a homomorphism from  $\mathbf{A}$  to  $\mathbf{B}$ . In this formalization, each relation of  $\mathbf{A}$  contains the tuples of variables that are constrained together, which are often called the *constraint scopes*, and the corresponding relation of  $\mathbf{B}$  contains the allowable tuples of values that the variable tuples may take.

It is well-known that the CSP, in its general formulation, is NP-complete; this general intractability has motivated a large and rich body of research aimed at identifying and understanding restricted cases of the CSP that are polynomial-time tractable. The restrictions that have been studied can, by and large, be placed into one of two categories, which—due to the homomorphism formulation of the CSP—have become known as *left-hand side* restrictions and *right-hand side* restrictions. From a high level view, left-hand side restrictions, also known as *structural restrictions*, arise from prespecifying a class of relational structures  $\mathcal{A}$  from which the left-hand side structure  $\mathbf{A}$  must come, while right-hand side restrictions arise from prespecifying a class of relational structures  $\mathcal{B}$  from which the right-hand side structure  $\mathbf{B}$  must come.

## Results

*Succinctly specified CSP.* With Martin Grohe, I investigated the complexity of the CSP where the right-hand side relations are specified succinctly. We have obtained results concerning the case where these relations are specified in a form similar to disjunctive normal form of propositional logic. In particular, we are able to characterize exactly those classes of relational structures  $\mathcal{A}$  for the left-hand side that yield tractability for this form. This characterization builds on previous work of Grohe [4]. This involves defining a measure of complexity for relational structures that falls inbetween the measure of treewidth [2] and the measure of generalized hypertree width. (See [3] for a survey.)

*Infinite Quantified CSP.* With Manuel Bodirsky, I investigated a generalization of the CSP called the quantified CSP (studied previously for instance in [1]). In this generalization, variables may be universally quantified in addition to being existentially quantified. We have initiated the study of the quantified CSP where the right-hand side structure is infinite, that is, has an infinite universe. In particular, we have started to investigate the case of *equality constraint languages*, which are constraint languages composed from the equality predicate as well as the boolean operators AND, OR, and NOT.

## Activities

- Attendance at some lectures of the EuroComb 2005 conference
- Participation in the logic seminars at HU
- Participation in the CGC annual workshop in Hiddensee
- Attendance and presentation at the Constraint Programming 2005 conference in Sitges, Spain
- Visit to the logic group of Jorg Flum in Freiburg, with colloquium presentation

### Contributions

- “Beyond Hypertree Width: Decomposition Methods Without Decompositions” lecture at the CGC annual workshop

- “The Complexity of Quantified Constraint Satisfaction” CGC colloquium
- “Decomposition Methods Without Decompositions in Constraint Satisfaction” lecture in the HU logic seminar

## References

- [1] Hubie Chen. The computational complexity of quantified constraint satisfaction. Ph.D. thesis, Cornell University, August 2004.
- [2] Victor Dalmau, Phokion G. Kolaitis, and Moshe Y. Vardi. Constraint satisfaction, bounded treewidth, and finite-variable logics. In *Constraint Programming '02*, LNCS, 2002.
- [3] Georg Gottlob, Nicola Leone, and Francesco Scarcello. A comparison of structural csp decomposition methods. *Artif. Intell.*, 124(2):243–282, 2000.
- [4] Martin Grohe. The complexity of homomorphism and constraint satisfaction problems seen from the other side. In *FOCS 2003*, pages 552–561, 2003.