Semester Report SS04 of Maike Buchin

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Supervisor:	Helmut Alt
Field of Research:	Computational Geometry
Topic:	Fréchet Distance of Triangulated Surfaces
PhD Student	in the program since May 2003

Field of Research and Results

I am interested in the computability of the Fréchet distance of triangulated surfaces.

Given two parameterized surfaces $f, g : [0,1]^2 \to \mathbb{R}^d,\ d \geq 2$, the Fréchet distance δ_F is defined as

$$\delta_F(f,g) := \inf_{\sigma:[0,1]^2 \to [0,1]^2} \max_{t \in [0,1]^2} ||f(t) - g(\sigma(t))||.$$

where the reparameterization σ ranges over all orientation preserving homeomorphisms. For curves, the Fréchet distance can be computed in polynomial time [2], in higher dimensions it is known to be NP-hard [3].

This semester I proved that the Fréchet distance of triangulated surfaces is semi-computable [1] in the computational model for real-valued functions defined by Weihrauch [4].

Theorem 1. *The Fréchet distance between two triangulated surfaces in $\mathbb{R}^d$, $d \geq 2$, is upper semi-computable, i.e. there is a Turing machine which outputs an infinite, monotone decreasing sequence of rational numbers converging to the Fréchet distance.*

This result implies the following corollary.

Corollary 1. *The decision problem for the Fréchet distance between triangulated surfaces is recursively enumerable.*

The resulting algorithm can also be applied to surfaces that are not triangulated but given by computable parametrizations. Then the algorithm outputs a sequence of rationals converging to the real value but which – in contrast to triangulated surfaces – does not converge from above or below, i.e., semi-computability is not achieved in this case.

There are several open questions remaining, in particular computability in the strong sense is still unknown. At the moment we do not see how to strengthen our method to show this, and are therefore trying other approaches to this problem and to related open questions.

Activities

Talks

- *Ambient Isotopic Approximation of 2-Manifolds*
  Noon Seminar of the TI-AG at the FU Berlin on September 9, 2004
- *On the Computability of the Fréchet Distance*
  4th CGC-Workshop in Stels, Switzerland, October 6, 2004
- *On the Computability of the Fréchet Distance Between Surfaces*
  Noon Seminar of the TI-AG at the FU Berlin on December 9, 2004

Attended lectures, seminars, and workshops

- *Monday Lectures and Colloquia* of CGC in Berlin
- *Noon Seminar* of the TI-AG at the FU Berlin
- *4th ICA Mountain Cartography Workshop*
  in Vall de Núria, Catalonia, Spain, September 30 – October 2, 2004
- *4th Workshop on Combinatorics, Geometry, and Computation*
  in Stels, Switzerland, October 4 – 7, 2004
- *Lecture Diskrete Geometrie* by Dr. Ivan Izmestiev at the FU Berlin, October 18 – December 13, 2004
- *2ter Doktoranden-Workshop des Instituts für Informatik der FU Berlin*, November 5 – 6, 2004
- *Lecture Advanced Probabilistic Methods*
  by Prof. Dr. Anand Srivastav at the FU Berlin, Januar 5 – 27, 2005
- *Seminar der Theoretischen Informatik (Mittagsseminar)*
  at the ETH Zurich (attended since February 3, 2005)

Preview

On January 31, 2005 I started a research visit at the ETH Zürich as part of my long term stay. At the ETH Zürich I am working together with Dr. Joachim Giesen. I plan to stay at the ETH Zürich until May 13, 2005.

In the next semester I will present my work at the European Workshop on Computational Geometry in Eindhoven in March and will attend the Spring school in Eindhoven. Also I am planning to attend the Spring School of CGC in June in Netzeband near Berlin.

References

[1] Helmut Alt and Maike Buchin. Semi-computability of the Fréchet distance between surfaces. In *Proc. 21st European Workshop on Computational Geometry*, Eindhoven, Netherlands, 2005. To appear.

[2] Helmut Alt and Michael Godau. Computing the Fréchet distance between two polygonal curves. *Internat. J. Comput. Geom. Appl.*, 5:75–91, 1995.

[3] Michael Godau. *On the complexity of measuring the similarity between geometric objects in higher dimensions.* PhD thesis, Freie Universität Berlin, Germany, 1998.

[4] Klaus Weihrauch. *Computable Analysis.* Texts in Theoretical Computer Science. Springer-Verlag, Berlin, Heidelberg, 2000.