Semester Report WS04/05 of Taral Guldahl Seierstad

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Supervisor(s):	Prof. Dr. Hans Jürgen Prömel
Topic:	Random graph processes
PhD Student	at the program since January 2004

Field of Research

I study random graphs and random graph processes. This semester I have in particular studied the min-degree random graph process, see [2]. Let $G_{\min}(n, 0)$ be the empty graph on n vertices, and for $M > 0$ let $G_{\min}(n, M+1)$ be obtained from $G_{\min}(n, M)$ by first choosing a vertex of minimum degree in $G_{\min}(n, M)$ uniformly at random, and then connecting it by an edge to another vertex chosen uniformly at random from the remaining vertices in $G_{\min}(n, M)$. We say that a property holds *asymptotically almost surely* if it holds with probability tending to one as n tends to infinity.

We set $t = M/n$, and we consider the graph $G_{\min}(n, tn)$. The statements in this paragraph hold asymptotically almost surely, unless otherwise stated. It was shown in [2] that there are constants h_1, h_2 and h_3, such that for $1 \leq i \leq 3$ when $t < h_i$ the minimum degree is smaller than i, while when $t > h_i$ the minimum degree is at least i. Furthermore it was proved that when $t < h_2$, the graph is disconnected, while when $t > h_3$ the graph is connected. When $h_2 < t < h_3$, the graph is connected with probability bounded away from zero and one.

In the ordinary random graph model $G(n, p)$ a random graph is obtained by taking the empty graph on n vertices and adding every possible edge independently with probability p. In $G(n, p)$ there is a *phase transition* occuring: when $p < \frac{1}{n}$, then the graph asymptotically almost surely contains no components with more than $O(\log n)$ vertices, while when $p > \frac{1}{n}$, the graph contains a *giant component* of linear size, plus a number of smaller components with $O(\log n)$ vertices.

We wanted to see if there is a similar phase transition in the min-degree graph process where the component structure changes dramatically during a short period of time. Modelling the growth of the components using a *branching process* (see [1]) we were able to prove that such a phase transition does indeed take place.

Results

Dr. Mihyun Kang (also at the Humboldt University Berlin) and I proved that there is a constant $h_g \approx 0.8607$ such that when $t < h_g$, the largest component in $G_{\min}(n, tn)$ asymptotically almost surely has $O(\log n)$ vertices, but when $t > h_g$, the graph asymptotically almost surely contains a giant component of linear size. Furthermore, if there are other components they are of order $O(\log n)$

Activities

- Attended weekly lectures and colloquia of the CGC.
- Attended weekly seminar of the research group *Algorithmen und Komplexität* at the HU Berlin. (12 November 2004 I held a talk titled *Restricted random graph processes.*)
- Attended the 4th Workshop on Combinatorics, Geometry, and Computation, 2004 in Hof de Planis, Stels, Switzerland, 4 to 7 October 2004.
- Attended Symposium Diskrete Mathematik 2004 at ETH Zürich, 7 and 8 October 2004.
- Attended Learn- & Workshop on Randomness, Geometry, and Counting at TU Berlin, 6 to 8 December 2004.
- Attending Doccourse "Modern Methods in Ramsey Theory" at the Charles University in Prague, 25 January to 28 February 2004.

Preview

I will continue to study the min-degree process, in particular we are interested in the distribution of small cycles, and in studying closer the phase transition. It would be interesting to find out how the graph behaves near the critical moment, namely when $t = h_g + o(1)$, and whether a double jump occurs, as in $G(n, p)$.

References

[1] Krishna B. Athreya and Peter E. Ney. *Branching processes.* Springer-Verlag, New York, 1972. Die Grundlehren der mathematischen Wissenschaften, Band 196.

[2] Mihyun Kang, Youngmee Koh, Tomasz Łuczak, and Sangwook Ree. The connectivity threshold for the min-degree random graph process. submitted.