Semester Report WS04/05 of Dirk Schlatter

Name:	Dirk Schlatter
Supervisors:	Prof. Hans Jürgen Prömel and Prof. Anusch Taraz
Field of Research:	Random Discrete Structures
Topic:	Planar Graphs
PhD Student	at the program since December 2002

Field of Research

The main part of my research is concerned with the following restricted random graph process: Starting from an empty graph on n vertices, in each step, choose a random edge (of K_n) and insert it into the present graph if it remains planar. The probability of an edge being inserted at a certain stage in this process is of course highly dependent on the previous choices, quite contrary to the situation in the standard random graph model.

The main conjecture is that, as the density evolves, this random planar graph process differs significantly from a uniformly random planar graph of the same (edge) density (see [3] and [2]), but they both share some interesting properties, e.g. they contain a.a.s. all planar subgraphs of constant size.

Together with Stefanie Gerke, Angelika Steger, and Anusch Taraz, I am currently investigating the early stages of the process, i.e. when the density of the random planar graph process is $1 + \epsilon$. We know that a.a.s. it is already connected, and found evidence which suggests that the number of edges which may be added in the next step should be almost linear. This would imply that the plane embedding is already quite rigid, which in turn should provide us with a way to prove the almost sure containment of small planar graphs.

For the first few weeks of the past semester, I have continued work on upper bounds for the (unrestricted) random graph $G_{n,p}$ to contain a planar subgraph of given density d. To improve the currently best known upper bound of $p = n^{1/(d+\epsilon)}$ ([4]), an approach using the second moment method on weighted indicator variables, similar to the one used e.g. in [1], seems promising.

Activities

Conferences and Workshops

- OCTOBER 4 – 7 CGC Annual Workshop in Stels (talk: The Random Planar Graph Process)

- OCTOBER 7 – 8 DMV Symposium on Discrete Mathematics in Zürich

- DECEMBER 6 – 8 Learn- and Workshop on *Randomness, Geometry, and Counting* at TU Berlin

Lectures and Seminars

- WEEKLY seminars of the Institute of Theoretical Computer Science at ETH Zürich (talk: Spanning Subgraphs of Random Graphs, November 16)

- WEEKLY lecture on *Satisfiability* at ETH Zürich

- NOVEMBER 29 lecture and colloquium of the CGC (talk: Spanning Subgraphs of Random Graphs)

Research stay

- NOVEMBER 22 – 26 with Anusch Taraz at TU München

Preview

I will continue to work on the random planar graph process described above. I plan to visit the ASZ Learn- and Workshop on *Random Graphs and Probabilistic Methods* (March 7 – 9 at HU Berlin) and the CGC Spring School on *Enumerative Combinatorics* (June 1 – 4 in Netzeband). After the end of my long-term stay at ETH Zürich in March, I will return to Berlin for the summer semester.

References

[1] D. Achliopotas and Y. Peres, *The Threshold for Random k-SAT is* $2k \log 2 - O(k)$, Journal of the AMS, 17, 947-973, 2004.

[2] S. Gerke, C. McDiarmid, A. Steger, and A. Weißl, *Random planar graphs with n nodes and a fixed number of edges*, Proceedings of the 16th ACM-SIAM Symposium on Discrete Algorithms (SODA'05), 999-1007, 2005.

[3] C. McDiarmid, A. Steger, and D.J.A. Welsh, *Random planar graphs*, Journal of Combinatorial Theory, Series B 93,187-205, 2005.

[4] D. Schlatter and A. Taraz, *Existence and construction of planar subgraphs*, preprint, 24 pages