Semester Report WS04/05 of Andreas Paffenholz

Name:	Andreas Paffenholz
Supervisor(s):	Günter M. Ziegler
Field of Research:	Discrete Geometry
Topic:	Flag Vectors of 4-Polytopes
PhD Student	at the program since April 2002

Field of Research and Results

Most of the time of this semester I spent – and will still spend – on writing up my thesis. This semester is my last in the graduate program, and I hope to have handed in my thesis by the end of March.

In November and December I finished the work on my recent paper on the application of the E-construction to products of polytopes, which I submitted just before Christmas [5]. I completed the proof of the following fact: Among the polytopes, that one obtains from the application of the E-construction to a product of two polygons, most have automorphisms of their face lattice that are not realisable in a geometric realisation of the polytope. More precisely, one can prove the following. Let $C_n \times C_m$ be a product of two polygons with m and n vertices. Let E_{mn} denote the CW sphere obtained from the application of the E-construction to this product (See [6] for details about the E-construction.). I proved that all CW spheres E_{mn} are polytopal, and that they have quite flexible geometric realisations. So for most of the automorphisms of their face lattice one can find a realisation such that there is an affine map realising this automorphism. However, if m, n are relatively prime and both at least 5, then there are also automorphisms that cannot be realised geometrically in *any* geometric realisation of the polytope. During the proof I construct an explicit example of such an automorphism. This proof of the polytopality of the spheres E_{mn} and the existence of non-realisable symmetries extends some previous results of Bokowski and Gevay. They gave combinatorial descriptions of the E_{mm}, but without proving polytopality. They also conjectured some symmetry properties of these [2, 3] which are verified by my construction.

My thesis will consist of three parts, that basically reflect my three papers [1], [5] and [6]. The second paper, which was a joint work with Günter M. Ziegler, has recently appeared in *Discrete and Computational Geometry*. The

first, with Anders Björner, Jonas Sjöstrand, and Günter M. Ziegler, has been accepted for publication in the same journal last summer. I have extended some of the topics of the last two papers for my thesis and worked out several new examples and some additional properties of the E_{mn} since the beginning of this year. In particular I have improved the computation of realisation spaces for small examples of these polytopes E_{mn}. In the course of this I have simplified several proofs of [6].

Beside writing my thesis I also work on a new approach to either find 2-cubical and 2-cocubical 4-polytopes or prove that they do not exist. This is a project that I have pursued for quite a long time by now together with Carsten Lange [4]. We managed to solve all cases of k-cubical and h-cubical d-polytopes except the case $k + h = d$. In the current attempt I want to adapt a software presently developed by Axel Werner that searches for 2-simple and 2-simplicial 4-polytopes by trying to find an "inverse shelling" of such polytopes. In the remaining weeks of the semester, I will probably concentrate on writing up my thesis, but I hope to have a useful version of this programme sometime in April.

Since this is my last semester in the programme, at this point I would like to thank all people that supported my work in the graduate programme "Combinatorics, Geometry, and Computation". These are in particular the speaker of the programme Helmut Alt and the coordinators Bettina Felsner and Andrea Hoffkamp. I would also like to thank my supervisor Günter M. Ziegler for his great support.

Activities

- Lectures and Colloquia of the CGC
- CGC Annual Workshop, Stels, Switzerland, October 4–7
- DMV Tagung, Heidelberg, September 12–17, 2004
- Kolloquium über Kombinatorik, November 12–13, 2004, Magdeburg

I gave talks at the following occasions:

- Talk on "New Polytopes from Products", September 14, 2004, DMV Tagung, Heidelberg

- Talk on "Polytopes from Products," October, 6, 2004, CGC Annual Workshop, Stels, Switzerland
- Talk on "Polytopes from Products," November 2004, Colloquium of the CGC, TU Berlin
- Talk on "Polytopes from Products," Magdeburg, November 2004, Kolloquium über Kombinatorik

References

[1] Anders Björner, Andreas Paffenholz, Jonas Sjöstrand, and Günter M. Ziegler, *Bier spheres and posets*, 2004, preprint, TU Berlin, 15 pages, arXiv:math.CO/0311356.

[2] Jürgen Bokowski, *personal communication*, September 2004.

[3] Gábor Gévay, *personal communication*, November 2004.

[4] Carsten Lange and Andreas Paffenholz, *Cubical and cocubical polytopes*, in preparation.

[5] Andreas Paffenholz, *New polytopes from products*, 2004, preprint, TU Berlin, 22 pages, arXiv:math.MG/0411092.

[6] Andreas Paffenholz and Günter M. Ziegler, *The E_t-construction for lattices, spheres and polytopes*, Discrete Comput. Geom. **32** (2004), no. 4, 601–621.