Semester Report WS04/05 of Esther Moet

Name:	Esther Moet
Supervisors:	Dr. René van Oostrum (Utrecht)
	Prof. Dr. Christian Knauer (Berlin)
Field of Research:	Computational Geometry
Topic:	Visibility Problems in Polygons and Terrains
PhD Student	at the program from October 2004 to February 2005

Field of Research

Visibility problems have received much attention in the field of Computational Geometry since the famous *Art Gallery Problem* was solved by Chvátal in 1975 [1]. The last two decades, many variations of this problem have been studied, not only in polygons in the plane, but also for scenes in 3-space. We say that two points p and q are mutually visible if the open line segment connecting the two points does not intersect the exterior of the polygon (in 2D), or the interior of any of the obstacles (in 3D). During my Marie Curie stay of four months at the Freie Universität Berlin, we addressed two different visibility problems, one in the plane, and one in three-dimensional space.

Results

Segment Witnesses (joint work with Prosenjit Bose, Herman Haverkort, Christian Knauer, and René van Oostrum)

A witness set is a set of objects W, located inside a polygon P, with the following property: if a set of points G sees all elements in W, then G is a guard set for P. In an earlier paper [2], we considered *point* witnesses and proved that (1) not every polygon admits a finite point witness set, (2) it can be decided in $O(n^2 \log n)$ time whether P is *finite witnessable*, and (3) if such a set exists, it can be constructed in $O(n)$ time.

Point (1) above encourages us to consider other witness sets than point sets. As every polygon P is trivially witnessable by a two-dimensional region, viz. by P itself, we consider segment witnesses. Since visibility of a point p from a line segment s can be either *weak*, at least one point on s sees p, or *strong*, all points on s see p, this leads to some reconsiderations of the definitions used in [2]. We showed that the boundary of a star-shaped polygon P is a segment witness set for P, but this does not hold for general polygons. This implies that, unlike in [2], witnesses in the interior of P might be necessary.

However, interior witnesses again lead to choices to be made in the employed definitions; a segment in the interior of a polygon can be visible from 360°, as opposed to the 180° for segments on the boundary of a polygon. Work on this problem is still in progress.

Edge Visibility Map (joint work with Christian Knauer and Marc van Kreveld)

During the second part of my stay, we bounded the combinatorial complexity of the visibility map of an edge in 3D scenes. This structure is defined as the subdivision of the points on the triangles of the scene into visible and invisible connected components. For a single point, it is well known that this structure has complexity $\Theta(n^2)$ [3]. Surprisingly, the visibility map for an edge *in the plane* can already have $\Omega(n^4)$ vertices, if the polygon is allowed to have holes [4].

We extended the lower bound of [4] to show the edge visibility map can be $\Omega(n^4)$ for polyhedral terrains and $\Omega(n^5)$ for general 3D scenes. Furthermore, we proved bounds on the shadow that is cast on different features of the 3D scene, when we consider the edge to be a light source. We showed that this shadow is $O(n^2)$ on a single edge, and $O(n^4)$ on one triangle. Putting these results together yields a bound of $\Theta(n^5)$ on the complexity of the visibility map of an edge in general 3D scenes. For terrains, we are confident to prove an upper bound of $O(n^{4+e})$ shortly, which is very close to the above mentioned lower bound.

Activities

Presentations:

- *Weak Region Intervisibility in Terrains*,
  Noon Seminar; November 18th, 2004
- *Guarding Art Galleries by Guarding Witnesses*
  CGC Colloquium; December 13th, 2004
- *The Visibility Map of an Edge in 3D Scenes*
  Noon Seminar; February 17th, 2005

Other:

- Participation in the 4th Workshop on Combinatorics, Geometry, and Computation in Stels, Switzerland (October 4-7, 2004)

- Succesful completion of the course *Kombinatorische Optimierung* (Combinatorial Optimization, Prof. Dr. Günter Rote, 7 ECTS)
- Attendance at the Lectures and Colloquia of the Graduate Program Combinatorics, Geometry and Computation (once a week)
- Attendance at the Noon Seminar of the Work Group Theoretical Computer Science of the Freie Universität Berlin (twice a week)

Preview

- A selection of the results that were obtained during my stay, and future results in the research that was initiated in this period, will be included in my PhD thesis, which is expected to be finished in September 2007.

References

[1] V. Chvátal. *A combinatorial theorem in plane geometry.* J. Combin. Theory Ser. B, 18:39–41, 1975.

[2] K.-Y. Chwa, B.-C. Jo, C. Knauer, E. Moet, R. van Oostrum & C.-S. Shin. *Guarding Art Galleries by Guarding Witnesses.* Technical Report UU-CS-2003-044, Utrecht University, 2003.

[3] M. McKenna. *Worst-case optimal hidden-surface removal.* ACM Trans. Graph., 6:19–28, 1987.

[4] S. Suri & J. O'Rourke. *Worst-case optimal algorithms for constructing visibility polygons with holes.* Proc. 2nd Annu. ACM Sympos. Comput. Geom., 14–23, 1986.