Semester Report WS04/05 of Oliver Klein

Name:	Oliver Klein
Supervisor:	Prof. Dr. Günter Rote
Field of Research:	Computational Geometry and Combinatorics
Topic:	Matching Shapes with a Reference Point
PhD Student	in the program since April 2003

Field of Research

The topic of my research can be summarized as the analysis of reference points and their usage for shape matching under classes of transformations, like translations, rigid motions (translations and rotations) and similarity operations (rigid motions and scalings). Let $\mathcal{K} \subset \mathbb{R}^d$ be a set of shapes and δ be a distance measure on $\mathcal{K}$. Then a δ-reference point for $\mathcal{K}$ is defined as a Lipschitz-continuous mapping $r : \mathcal{K} \to \mathbb{R}^d$ that is equivariant with respect to the considered set of transformations $\mathcal{T}$. The Lipschitz-continuity can be expressed by

$$\forall A, B \in \mathcal{K} \ : \ ||r(A) - r(B)|| \leq c \cdot \delta(A, B).$$

In this context, the Lipschitz-constant c is called the quality of the reference point.

The first problem arising is to find reference points for different sets of shapes $\mathcal{K}$ and distance measures δ. The second challenge is to find ways of using these reference points to construct approximation algorithms based on them. In particular this means to find algorithms which find a transformation Φ^*, such that

$$\delta(A, \Phi^*(B)) \leq \alpha \cdot \min_{\Phi \in \mathcal{T}} \delta(A, \Phi(B)),$$

where $\alpha \in \mathbb{R}^+$ is some constant independent from the concrete choice of A and $B \in \mathcal{K}$. In this context α is called the approximation ratio of the algorithm.

The research is based on papers by Alt, Behrends and Blömer ([1]) and by Alt, Aichholzer and Rote ([2]). In particular, in [2] the case of $\mathcal{K} = \mathcal{C}^2$, the set of compact convex subsets of $\mathbb{R}^2$, and δ as the Hausdorff-Distance δ_H is considered. This distance measure is defined as the smallest ε such that the Euclidean distance from every point of A to its nearest point of B

is at most ε and vice versa. Algorithms to determine the optimal transformation minimizing the Hausdorff-Distance under translations, rigid motions and similarity operations are known, but unfortunately the running time of these algorithms is not satisfactory for most applications. The authors of [2] develop approximation algorithms using reference points for matching under translations, rigid motions and similarity operations with approximation ratios $c+1$, $c+1$ and $c+3$, respectively. Furthermore, it is shown that the Steiner point (Steiner Curvature Centroid) is a δ_H-reference point for $\mathcal{C}^2$ of quality $\frac{4}{\pi}$. It is also shown, that this quality is optimal, which means that there cannot exist any δ_H-reference point with a smaller Lipschitz-constant. This is shown using strong functional-analytic tools and the axiom of choice. Therefore the proof is not constructive.

Summarizing the lower bound on the quality of a reference point of $\frac{4}{\pi}$ and the upper bound of the algorithm using reference points of quality $c+1$ with respect to translations, it seems reasonable that there are shapes $A_1, A_2, \ldots$ which cannot be matched in a way that

$$\forall i \neq j \ : \ \delta_H(A_i + t_i, A_j + t_j) \leq (1 + \frac{4}{\pi} - \beta) \cdot \delta_H^{opt}(A_i, A_j), \qquad (1)$$

where $\delta_H^{opt}(A, B)$ is the optimal Hausdorff-Distance under translations, $\beta \in \mathbb{R}^+$ is any constant and $t_i \in \mathbb{R}^2$ are translation vectors. Observe that under these assumptions the vectors t_i can be interpreted as the reference points of the given shapes. Unfortunately, those shapes A_i, or even their existence, are not known so far. More knowledge about the nature of these shapes would lead to a better understanding of the approximation algorithms and, eventually, to even better algorithms.

Our most recent, more theoretical approach is based on the non-convex shapes given in [2] showing a lower bound for the quality of the Steiner Point. We could use them to create a small set of three shapes proving a non-convex lower bound for the approximation ratio of the translation algorithm of 1.5.

In October 2004, in the early beginning of my time in Utrecht, Remco Veltkamp and I had the idea of applying the reference point approach to weighted point sets with respect to the Earth Mover's Distance (EMD). The Earth Mover's Distance on weighted point sets is a useful distance measure for e.g. shape matching and colour-based image retrieval, see [3], [4] and [5] for more information. The EMD between two weighted point sets A and B can be interpreted as follows: Imagine the weighted points of A as piles of earth and those of B as holes. Basically, the EMD is the physical work needed, to move the piles of earth of A into the holes of B. Until now we got a couple of interesting results on this. First, we have shown that there is a reference point for weighted point sets with equal total weight, namely the centers of mass of these sets. The quality of this reference point is 1. Second, we have shown that there is no reference point for weigthed point sets with unequal total weights. Furthermore we have developed approximation algorithms for translations, rigid motions and similarity operations with approximation ratios $c+1$, $2(c+1)$ and $4(c+1)$. The running time of these algorithms is $O(T^{EMD})$ for translations and $O(n^2 T^{EMD})$ for rigid motions and similarity operations. Here, $O(n)$ is the number of points of A and B and T^{EMD} denotes the time needed to compute the EMD between those two sets. Additionally, the results are independent of the dimension of the weighted point set and therefore the results are widely applicable.

We have also worked on the Proportional Transportation Distance (PTD) and showed that the center of mass in this case is a reference point for weighted point sets with arbitrary total weights. Also the other results proved for the EMD can easily be adapted for the PTD.

The results have been accepted for EWCG 2005 in Eindhoven and a technical report has been published, see [6].

Future Work

There are still a number of open problems. I would like to finish the work started in Utrecht: First of all, the running time and approximation ratio of the algorithms for rigid motions and similarity operations under the EMD do not seem to be optimal. I have some promising ideas on how to improve on this.

Second, the EMD for weighted point sets is an instance of a more general definition of the EMD for arbitrary subsets of $\mathbb{R}^d$, the so-called Monge-Kantorovich-Distance, see [7] for more details. We would like to generalize the reference point method to work on subsets of $\mathbb{R}^d$ with respect to this distance measure. The Monge-Kantorovich-Distance is some combination of the Hausdorff-Distance and the Distance of the Area of Symmetric Difference. Therefore I hope to get more insight into the behaviour of the approximation algorithms with respect to the Hausdorff-Distance. Finally, I hope to get back where I started my research, namely finding a lower bound for matching under translations with respect to the Hausdorff-Distance. At this point I want to use the gained insight to extend the discussion and eventually finding better approximation algorithms.

As mentioned above, the non-convex lower bound on the quality of the Steiner point could be used to get a non-convex lower bound on the approximation ratio of the translation algorithm. Lately, we found a new convex lower bound on the quality of the Steiner point. We would like to use these shapes proving the bound to get a lower bound on the approximation ratio of the approximation algorithms.

While reading a report by Gerald Weber [8] my attention fell on a slightly stronger definition of regular reference points and some open problems connected to it. In this definition of regular reference points the set of allowed transformations to create the approximation is reduced. Therefore a general reference point need not be regular or it is regular but with a worse approximation ratio. However, up to now there is no general reference point known, that is not a regular reference point with the same approximation ratio. It may be interesting to find such a reference point.

Activities

Talks

- *Matching Point Sets in three Dimensions*
  Noon Seminar of the TI-AG at FU Berlin on July 22., 2004

- *The Steiner Point as a Reference Point: A Convex Lower Bound*
  Noon Seminar of the TI-AG at FU Berlin on September 30., 2004

- *Lower bounds for the Quality of Steiner Points and Shape Matching with Reference Points*
  4th Workshop on Combinatorics, Geometry and Computation in Stels, Switzerland, October 5., 2004

- *Reference Points and their Applicatians for Shape Matching*
  Colloquium of the Center for Geometry, Imaging and Virtual Environments (GIVE) at Universiteit Utrecht, October 28., 2004

Attended events

- *Monday Lectures and Colloquia* of CGC in Berlin

- *Noon Seminar* of the TI-AG at FU Berlin

- *4th Workshop on Combinatorics, Geometry and Computation* in Stels, Switzerland, October 4.-7., 2004

Long Term Exchange

- Research Stay at Universiteit Utrecht, Institute of Information and Computing Science, Center for GIVE, Workgroup of Prof. Dr. Mark Overmars, Supervisor Dr. Remco Veltkamp, October, 2004 - March, 2005.

Preview

- Talk in the Colloquium of the Center for Geometry, Imaging and Virtual Environments (GIVE) at Universiteit Utrecht, February 24., 2005
- Attend the Spring School on Computational Geometry at Technische Universiteit Eindhoven, March 7.-8., 2005
- Talk at 21th European Workshop on Computational Geometry at Technische Universiteit Eindhoven, March 9.-11., 2005
- Attend the Spring School on Enumerative Combinatorics, June 1.-4., 2005

References

[1] H. Alt, B. Behrends, J. Blömer: 'Approximate matching of polygonal shapes', Proc. 7th Ann. Symp. on Comp. Geom., 1991, 186-193

[2] O. Aichholzer, H. Alt, G. Rote: 'Matching Shapes with a Reference Point', in Int. J. of Comp. Geom. and Appl., Volume 7, pages 349-363, August 1997

[3] S. Cohen. Finding Color and Shape Patterns in Images. PhD thesis, Stanford University, Department of Compute Science, 1999.

[4] P. Giannopoulos, R. Veltkamp. A pseudo-metric for weighted point sets. In *Proc. 7th European Conf. Comp. Vision*, LNCS 2352, pages 715–731, 2002.

[5] K. Graumann, T. Darell. Fast contour matching using approximate Earth Mover's Distance. In *Proc. of the IEEE Conf. Comp. Vision and Pattern Recognition*, LNCS 2352, pages I: 220–227, 2004.

[6] O. Klein, R. C. Veltkamp. Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points. Technical Report UU-CS-2005-003, http://ftp.cs.uu.nl/pub/RUU/CS/techreps/CS-2005/2005-003.pdf, 2005.

[7] S. Rachev. The Monge-Kantorovich mass transference problem and its stochastic applications. In *Th. of Prob. and Appl.*, 29, pages 647–676, 1985.

[8] G. Weber: 'The Centroid is a Reference Point for the Symmetric Difference in d Dimensions', Technical Report UoA-SE-2004-1, The University of Auckland, 2004