Cornelia Dangelmayr, Semester Report WS04/05

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Field of Research:	Graph Theory
Topic:	Intersection Graphs and Graph Classes
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Field of Research

The intersection graph $\Omega(F)$ of a family of geometric objects in the plane $F = \{S_1, .., S_n\}$, S_i of type S, is the graph having F as vertex set with S_i adjacent to S_j if and only if $i \neq j$ and $S_i \cap S_j \neq \emptyset$. A graph $G = (V, E)$ is an intersection graph of sets of type S if there exists a family F such that $G \cong \Omega(F)$. As we restrict the sets to convex objects in the plane, the intersection graphs that arise can be used as an instrument to characterize and classify graphs. On one hand it is an interesting question which graphs can be described by intersection graphs of certain geometric objects. And on the other hand one can address himself to the task of relating graph theoretic parameters to geometric conditions such that the graphs can be assigned to classes of intersection graphs of certain objects according to their graph theoretic values.

In view of descriptions of graphs as intersection graphs of certain types of convex objects in the plane there do exist several results and construction algorithms for different classes of graphs. A well studied class of graphs are planar graphs PG, although there are still open problems, for example in the case of intersection graphs of 1-dimensional objects in the plane.

As an example of relations between intersection graphs and classical classes of graphs it is known, that interval and tolerance graphs, a generalization of interval graphs, are perfect graphs. So do chordal graphs, and while interval graphs do also belong to chordal graphs, the classes of tolerance and chordal graphs are different. Another aspect of interval graphs is the fact that they are cocomparability graphs. In this respect questions emerge like if comparability parameters of graphs G_P correspond to interval parameters like interval or track number of G_P or $\bar{G_P}$, or if there are tolerance graphs which are not cocomparability graphs [11].

Results

A well known class of intersection graphs is of families of line segments in the plane $\Omega(LS_k)$ where the line segments are parallel to at most k directions and no two parallel and no three intersecting ones do have a common point. If a graph G is an intersection graph of such line segments, we know immediatly that $k \geq \chi(G)$. In the case of planar graphs, it is known that bipartite planar graphs can always be represented by grid intersection graphs $\Omega(LS_2)$, showed by Hartmann et alt. in [1], and triangle-free planar graphs as $\Omega(LS_3)$, showed by Castro et alt. in [3]. I was able to formulate a construction that yields a representation of outerplanar graphs OPG as intersection graphs of line segments parallel to at most $k = 3 = \chi(OPG)$ directions $\Omega(LS_3)$. Motivated by this a representation for series parallel graphs as $\Omega(LS_3)$ was found in between by Kara. The general question if there exists a $k \in \mathbb{N}$ such that planar graphs are intersection graphs of line segments parallel to at most k directions $\Omega(LS_k)$ remains an open problem, and so does the question if all 3-colorable planar graphs are $\Omega(LS_3)$. At the moment I'm looking at planar graphs that are Eulerian.

On the other hand I was able to build graphs that are not representable as intersection graphs of line segments in the plane for any k. Those graphs will be denoted be $fb(LS_k)$. I define a complete subdivision of the edge set of a graph to be the operation where every edge uv is replaced by a node w of degree 2 and two edges uw, wv connecting w with the separated vertices u and v. The complete subdivisions of K_5 and $K_{3,3}$ are two examples of minimal graphs that are not representable as intersection graphs of line segments parallel to at most k directions for any k. It seems to be natural that there are more graphs F with crossing number $cr(F) \geq 1$ that give suchlike forbidden subgraphs for intersection graphs of objects in the plane.

I also worked on representations of planar graphs as track interval graphs. It is known that outerplanar graphs $G \in OPG$ are track interval graphs with track number $t(G) \leq 2$ [7]. In general this number is bounded by the caterpillar arboricity $c_\alpha(G)$, the minimal number of caterpillar forests the set of edges of a graph G can be partitioned in. The track number of planar graphs is not known, but $i(PG) = 3 \leq t(PG) \leq c_\alpha(PG) \leq st_\alpha(PG) = 5$, where $i(PG)$ is the interval number and $st_\alpha(PG)$ the star arboricity of the class of planar graphs. In the case of outerplanar graphs the upper bound of $c_\alpha(OPG) = 3$, so it must not be a sharp bound. Investigating a possible alternative for an upper bound for the track number of some special graphs, we found out that there is no general relation between the dimension $dim(P)$ of a poset P and the track number $t(\bar{G_P})$ of its cocomparability graph $\bar{G_P}$.

I will further concentrate on characterizations and relations between different classes of graphs, graph theoretic parameters and geometric restrictions resulting from the objects of the intersection graphs. The classification of graphs not representable as intersection graphs of certain objects in the plane, like line segments $\Omega(LS_k)$, also seems to be promising.

Activities

- I attended the Monday Lectures and Colloquia of the CGC.
- I participated in the workshop at Stels, October 4th to 7th 2004.
- I attended the DMV Symposium in Zürich, October 8th/9th 2004.
- I gave a talk at the weekly seminar of Prof. Aigner.

Preview

- Spring School 2005 on Enumerative Combinatorics in June

References

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