Semester Report WS04/05 of Kevin Buchin

Name:	Kevin Buchin
Supervisor:	Günter Rote
Field of Research:	Computational Geometry
Topic:	Geometric Structures on Point Sets, Probabilistic Analysis of Geometric Algorithms
PhD Student	in the program since May 2003

Field of Research and Results

This semester I continued working on an algorithm for constructing the Delaunay triangulation of a point set by inserting them in the order given by a space-filling curve [2]. Currently, I am visiting the group of Prof. Emo Welzl and am working with Dr. Joachim Giesen on flow complexes [4, 3].

In the previous semester, I had designed and analyzed on an algorithm for constructing the Delaunay triangulation of a point set. The algorithm uses a *biased randomized insertion order* [1], i. e. it assigns points to rounds of increasing size at random. The points of a round are inserted along a space-filling curve. To achieve a linear bound on the running time for uniformly distributed points in a square, different point location schemes are used depending on the position of a point. In this semester, I analyzed the structure of the Delaunay triangulation of uniformly distributed points near the boundary. With this I proved that the above algorithm remains linear when all points are located by *walking*, i. e. a local traversal of the data structure for the triangulation.

Theorem 1 *Using a biased randomized insertion order and, in each round, walking along a Lipschitz-*$1/2$*, bi-measure preserving space-filling curve, the incremental construction algorithm runs in linear expected time for points distributed independently and uniformly in a bounded, convex area.*

The algorithm runs also in higher dimensions but the analysis of the running time does not carry over to this case. It is known that the Delaunay triangulation can be constructed in expected linear time for points distributed uniformly at random in a d-ball.With a different point location scheme than above I proved that this can be done by an incremental construction algorithm. For this the points are inserted in random order. The algorithm maintains a dynamic bucketing scheme. This allows to find the nearest neighbor

in the triangulation for a new point in constant expected time using spiral search.Now a d–simplex incident to the nearest neighbor is found which conflicts with this point. From this simplex all conflicting d–simplices are found.

Theorem 2 *Using spiral search to find the nearest neighbor and finding an incident conflicting d–simplex, the incremental construction algorithm runs in linear expected time for points distributed independently and uniformly in a d–dimensional bounded convex open region for which the expected complexity of the Delaunay triangulation is linear. In particular, this is the case for the unit d–ball.*

Activities

Talks

- *Intersecting Delaunay Triangulations near the Boundary*
  Noon Seminar of the TI-AG at the FU Berlin on September 2, 2004
- *Illustrating Terrains using Direction of Slope and Lighting*
  4th ICA Mountain Cartography Workshop in Vall de Núria, Catalonia, Spain, on October 1, 2004
- *Locating Point Sets in Delaunay Triangulations*
  4th Workshop on Combinatorics, Geometry, and Computation in Stels, Switzerland, on October 5, 2004
- *Inkrementelle Konstruktion der Delaunay Triangulierung von zufälligen Punkten*
  2ter Doktoranden-Workshop des Instituts für Informatik der Freien Universität Berlin on November 6, 2004
- *A Collection of Linear Expected Time Delaunay Triangulation Algorithms*
  Noon Seminar of the TI-AG at the FU Berlin on November 30, 2004

Attended events

- *Monday Lectures and Colloquia* of CGC in Berlin
- *Noon Seminar* of the TI-AG at the FU Berlin

- *4th ICA Mountain Cartography Workshop* in Vall de Núria, Catalonia, Spain, September 30 – October 2, 2004
- *4th Workshop on Combinatorics, Geometry, and Computation* in Stels, Switzerland, October 4 – 7, 2004
- *Lecture "Diskrete Geometrie"* by Dr. Ivan Izmestiev at the FU Berlin, October 18 - December 13, 2004
- *2ter Doktoranden-Workshop des Instituts für Informatik der Freien Universität Berlin*, November 5 – 6, 2004
- *Learn- & Workshop "Randomness, Geometry, and Counting"* at the TU Berlin, December 6 – 8, 2004
- *Lecture "Advanced Probabilistic Methods"* by Prof. Dr. Anand Srivastav at the FU Berlin, Januar 5 – 27, 2005
- *Seminar der Theoretischen Informatik (Mittagsseminar)* at the ETH Zurich (attended since February 1, 2005)

Preview

Currently, I am working with Dr. Joachim Giesen on a general algorithm for computing the (unweighted and weighted) flow complex and on a bound on the complexity of the flow complex. In March, I will present my work on incremental construction algorithms at the European Workshop on Computational Geometry and attend the Spring School in Eindhoven. In June, I am planning to attend the spring school of CGC.

References

[1] N. Amenta, S. Choi, and G. Rote. Incremental constructions con BRIO. In *Proc. 19th Annual Symposium on Computational Geometry*, pages 211–219. ACM Press, 2003.

[2] K. Buchin. Incremental construction along space-filling curves. In *Proc. 21st European Workshop on Computational Geometry*, Eindhoven, Netherlands, 2005. To appear.

[3] J. Giesen and M. John. Computing the weighted flow complex. In *Proc. 8th International Fall Workshop Vision, Modeling, and Visualization*, pages 235–243, 2003.

[4] J. Giesen and M. John. The flow complex: a data structure for geometric modeling. In *Proc. 14th ACM-SIAM Symposium on Discrete Algorithms*, pages 285–294, 2003.