Scientific report of Andreas Westerlund

Name:	Andreas Westerlund
Supervisor(s):	Martin Grötschel and Ralf Borndörfer
Field of Research:	Combinatorial Optimization
Topic:	Vehicle routing problems with heterogeneous fleet
PhD Fellow:	at the program from November 2003 to April 2004

Field of Research and results

Given a set of vehicle types, distinguished by capacity and fixed cost, and a set of customers with known demands, we study a vehicle routing problem that can be described as follows. Decide which fleet of vehicles to use (that is, how many vehicles of each type) and construct routes for the vehicles so that all customers are served. Each customer must be visited by exactly one vehicle, and a capacity constraint and a time constraint associated with each vehicle must be fulfilled. It is allowed to use the same vehicle more than once during a day.

The optimization problem described above originates from the company DaimlerChrysler, which has provided us with real world data. The data covers the operation of a delivery service over a time horizon of 30 days, involving up to 303 customers and seven vehicle types of capacity varying from 1030 to 13025 kg, i.e. the Daimler Chrysler problems are large-scale heterogeneous vehicle routing problems. DaimlerChrysler developed a good heuristic, that can be used in order to compute an upper bound on the optimal objective value. In our project, we are developing decomposition methods based on a set partitioning formulation, where each column corresponds to the route(s) performed by a given vehicle during the day. There exist an enormous number of potential columns; they are not treated explicitly. Instead, column generation is performed. In one approach for a column generation problem, we extend the concept of a *q-path*, introduced in Christofides et al (1981), to fit in our context. Our aim is to compute strong lower bounds for the problem under consideration.

We have also studied a more restricted vehicle routing problem. For this version, a heterogeneous fleet of vehicles to use is given, and a vehicle can not be re-used. Moreover, there is no time restriction, only capacity restrictions. For this problem, an algorithm has been developed that can produce a strong lower bound on the optimal objective value. The original formulation has an exponential number of constraints. This large set of constraints is not treated explicitly. Instead, the general idea can be described as "relax-and-cut"; we start by using only a small number of constraints. New constraints are identified as they violate a Lagrangian subproblem (a k-tree problem) solution, and these constraints are then taken into account in a Lagrangian dual objective function. Our implementation can be viewed as a generalization of the lower bounding procedure presented in Fisher (1994) . He studies the homogeneous case, that is, when all vehicles have the same capacities.

Activities

- Attendance to the Lectures and Colloquium of the CGC program (once a week)
- One talk at the Colloquium of the CGC program ("The Heterogeneous Fleet Vehicle Routing Problem")
- Attendance to seminars at ZIB (once every two week)
- One talk at a seminar at ZIB ("Two generalizations of the Held and Karp 1-tree problem")
- Participation in the conference "Combinatorial Optimization 2004", Lancaster University, Lancaster, UK, 28-31 March, 2004.
- Cooperation with Stefan Gnutzmann, DaimlerChrysler Research, on the solution of large-scale heterogeneous fleet vehicle routing problems.

Preview

- Continue work to improve qualities of lower bounds with Stefan Gnutzmann and Ralf Borndörfer.
- Pursue an alternative column generation approach in a Diplom Arbeit at TU/ZIB.
- We are working on a publication on our results on lower bounds for heterogeneous vehicle routing problems with time and capacity constraints.

- The results of this project will be included in a chapter in my PhD thesis, planned to be completed by the end of this year.

Literatur

[1] N. Christofides, A.Mingozzi, and P. Toth. *Exact algorithms for the vehicle routing problem, based on spanning trees and shortest path relaxations*, Mathematical Programming, 20:255-282, 1981.

[2] M. Fisher, *Optimal solution of vehicle routing problems using minimum k-trees*, Operations Research, 42:626-642, 1994.