Semester Report WS03/04 of Maike Walther

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Field of Research:	Computational Geometry
Topic:	Fréchet distance of triangulated surfaces
PhD Student	in the program since May 2003

Field of Research

This semester, I continued my work on the Fréchet distance as a similarity measure for (parametrised) discrete geometric objects. In dimension 1 (of the parameter space), i. e. for polygonal curves, the Fréchet distance is known to be computable in polynomial time [AG95]. In higher dimensions, the decision problem of the Fréchet distance is NP-hard [God98]. My current research interest lies in the computability of the Fréchet distance in dimension 2 (of the parameter space), i. e. of triangulated surfaces.

One approach to showing the computability is by an arbitrary close discrete approximation. Given parametrized curves or surfaces

$$f, g : [0,1]^d \to \mathbb{R}^n \textit{ continuous},\ d \in \{2,3\},\ n \geq d$$

their Fréchet distance is defined as

$$\delta_F(f,g) := \inf_{\sigma:[0,1]^d \tilde{\to} [0,1]^d} \sup_{x \in [0,1]^d} ||(f(x) - g(\sigma(x)))||$$

where $\sigma : [0,1]^d \tilde{\to} [0,1]^d$ denotes orientation preserving homeomorphisms. In a discrete approximation, instead of taking the infimum over all homeomorphisms, we want to take the minimum over a finite set of (discrete) functions.

The motivation for this is that the objects in consideration are discrete. That is, we assume f and g to be simplicial, which means that their parameter spaces are the underlying spaces of d-dimensional simplicial complexes, such that f and g are affine on the simplices. In particular, f and g can be finitely described by their values on (finitely many) simplices.

If f and g act on the underlying spaces of the simplicial complexes K and L, respectively, we want to define the discrete Fréchet distance as

$$\delta_{dF}(f,g) := \min_{h:\cup K \to \cup L} \max_{\alpha \in \cup K} \delta(f(\alpha),\, g(h(\alpha)))$$

where h fulfills certain properties and $\delta(f(\alpha),\ g(h(\alpha)))$ is defined as a distance between the image of the simplex α in K under f and the image of the simplex $h(\alpha)$ in L under g.

For this discrete Fréchet distance we want to show

$$\delta_F(f,g) \ \leq \ \delta_{dF}(f,g) \ \leq \ \delta_F(f,g) + c(K,L) \tag{1}$$

where c depends on the fineness of the simplicial complexes K, L (i. e. the length of their longest edge) and c goes to 0 with the fineness of K and L. With this, we can approximate the Fréchet distance arbitrarily close by the discrete Fréchet distance, by refining the triangulations of f and g.

For polygonal curves such a discrete Fréchet distance has been described by Eiter and Mannila [EM94]. Because the parameter spaces of curves are 1-dimensional, and therefore linearly ordered, the homeomorphisms are exactly the continous, strictly monotone functions. For the discrete Fréchet distance, h ranges over all discrete, monotone functions and (1) holds.

In dimension 2, i. e. for surfaces, the parameter spaces $[0,1]^2$ are not linearly ordered. Therefore, a discrete approximation is not as straightforward, but an open question, on which I am currently working.

Activities

Talks

- *Multivariate Asymptotic Notation: A Flaw and A Fix*, Noon Seminar of the TI-AG at the FU Berlin, 9.9.2003
- *On the Fréchet distance of curves and surfaces*, Annual Workshop of CGC in Neustrelitz, 30.9.2003
- *Godau's Proof of the NP-hardness of the decision problem for the Fréchet metric in higher dimensions*, Noon Seminar of the TI-AG at the FU Berlin, 13.11.2003 and 20.11.2003

Attended events

- *Monday lectures and colloquia* of CGC in Berlin
- Lecture *Selected topics in Computational Geometry* by Dr. Christian Knauer at the FU Berlin

- Lecture *Convex Geometry* by Dr. Ivan Izmestiev at the FU Berlin
- *Noon Seminar* of the TI-AG at the FU Berlin
- *4th Max-Planck Advanced Course on the Foundations of Computer Science* (ADFOCS 2003) in Saarbrücken, 8.-12.9.2003
- Annual Workshop of CGC in Neustrelitz, 28.9.-1.10.2003
- Fall School *Computational Geometry* in Neustrelitz, 2.-4.10.2003
- Learn- & Workshop *Randomness in the design and analysis of algorithms* at the Konrad-Zuse-Zentrum in Berlin, 18.-20.10.2003

Other activities

- Research visit with Dr. Mario Cousta Sousa at the University of Calgary, 16.7.-7.8.2003
- Referee for SODA 2003

Preview

Block courses at the Charles University in Prague, 26.1. - 2.3.2004

References

[AG95] Helmut Alt and Michael Godau. Computing the Fréchet distance between two polygonal curves. *Internat. J. Comput. Geom. Appl.*, 5:75–91, 1995.

[EM94] Thomas Eiter and Heikki Mannila. Computing Discrete Frechet Distance. Technical Report CD-TR 94/65, Christian Doppler Laboratory, 1994.

[God98] Michael Godau. *On the complexity of measuring the similarity between geometric objects in higher dimensions.* PhD thesis, Freie Universität Berlin, Germany, 1998.