Semester Report WS03/04 of Dirk Schlatter

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Field of Research:	Random Discrete Structures
Topic:	Planar Graphs
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Field of Research

Around the beginning of the past semester, I started work on random planar graphs. Together with A. Taraz, I have been studying *triangulation processes*: Starting from an empty graph on n vertices, in each step, choose a random edge (of K_n) and insert it into the present graph if it remains planar. One central question is whether the random triangulation thus obtained differs from the uniform random triangulation, and if so, in what way. The probability of an edge being inserted at a certain stage in this process is of course highly dependent on the previous choices, quite contrary to the situation in the standard random graph model. It therefore comes as no surprise, that the usual probabilistic methods are difficult to apply to this case. So far, the only exact result concerns the restricted case when we start from a Hamilton cycle and only triangulate one of its two regions.

Recently, I have also become interested in the enumeration problem for planar graphs: the size of the set $\mathcal{P}_n$ of all (labelled) planar graphs is still not known exactly — the currently best lower and upper bounds are

$$n!(26.18)^{n+o(n)} \leq |\mathcal{P}_n| \leq (32.16)^{n+o(n)},$$

see [1] and [2], respectively. With a new enumeration idea by A. Taraz, based on spanning trees, we are currently working to improve these bounds. Apart from the obvious connections between properties of triangulation processes and the enumeration of planar graphs, there are also quite subtle ones: in both settings, the following question arises: given a Hamilton cycle, in how many ways can it be triangulated?

As far as my "old" problem of determining the threshold for a random graph to contain a spanning planar subgraph of some given density is concerned, I have reached the conclusion that it might well be as difficult as the long outstanding problem to determine the threshold of a complete vertex covering by disjoint triangles. Both problems investigate the occurence of certain spanning subgraphs of random graphs, and contrary to the containment problem for small subgraphs, very little is known in this area. Thus it seems worthwhile to study the relevant literature (see [3],[5] and notably the connection to perfect matchings in random 3-uniform hypergraphs [4]).

Activities

Conferences and Workshops

- SEPTEMBER 28 – OCTOBER 1 CGC Annual Workshop in Neustrelitz (talk: *Triangulations processes*)
- OCTOBER 2 – 4 CGC Fall School on *Computational Geometry* in Neustrelitz
- OCTOBER 18 – 20 ASZ Learn- and Workshop on *Randomness in the design and analysis of algorithms* in Berlin

Lectures and Seminars

- WEEKLY lectures and colloquia of the CGC (talk: *Random triangulations*, October 27)
- WEEKLY seminar of the research group *Algorithmen* at the HU Berlin
- WEEKLY seminar on *Combinatorics and its applications* at the HU Berlin

Preview

In the forthcoming weeks, I will continue the work on improving the known bounds for $|\mathcal{P}_n|$. In April and May, I intend to participate in the block courses *Convex Polytopes* (by Günther M. Ziegler) and *Random generation and approximate counting* (by Volker Kaibel).

References

[1] A. Bender, Z. Gao and N.C. Wormald, *The number of labeled 2-connected planar graphs*, The Electronic Journal of Cmbinatorics, 9(1):R43, 2002.

[2] N. Bonichon, C. Gavoille and N. Hanusse, *An information–theoretic upper bound of planar graphs using triangulation*, in 20^{th} Annual Symposium on Theoretical Aspects of Computer Science (STACS), volume 2607:499-510 of Lecture Notes in Computer Science, 2003. Springer-Verlag.

[3] A. Frieze and S. Janson, *Perfect matchings in random s-uniform hypergraphs*, Random Structures and Algorithms, 7:41-57, 1995.

[4] J.H. Kim, *Perfect matchings in random uniform hypergraphs*, submitted.

[5] M. Krivelevich, *Triangle factors in random graphs*, Combinatorics, Probability & Computing 6:337-347, 1997.