Semester Report WS03/04 of Martin Kutz

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Topic:	Games on Hypergraphs
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Positional Games

In a *weak positional game*, two players, called *Maker* and *Breaker*, alternately color the vertices of a hypergraph in their respective colors, Maker trying to create a monochromatic edge of his color—a Maker win. Breaker tries to prevent this and wins if at the end of the game, when all vertices are colored, no edge is monochromatic of Maker's color. (Note that Breaker does not win by completing some edge in his own color.) Such games have been studied, for example, in and [2] and [4].

Deciding whether an arbitrary given hypergraph can be won by Maker is PSPACE-complete. More precisely, in [6] Schaefer proves that this problem is PSPACE-complete for hypergraphs of rank 11, where the rank of a hypergraph is defined as the maximum number of vertices in an edge.

Already two semesters ago, I solved the problem of identifying winners among hypergraphs of rank 3 that are also almost-disjoint, which means that any two edges may share at most one common vertex. For such hypergraphs a winning strategy can be found in polynomial time, if one exists. This complexity result comes with a nice classification of winners and losers in terms of contained substructures.

Conway's Angel Problem

We consider the following game with a chess king on an infinite checker board: The king moves across the board, according to the usual chess rules and we want to trap him by destroying squares of the board. Precisely, in each move the king steps onto an undamaged square adjacent to his current position and we in turn delete an arbitrary square from the board Can you trap the king, that is, do you have a strategy such that at some point the king cannot move any more; or is he able to run on forever?

Berlekamp showed that you can trap him, but the problem gets amazingly difficult if you increase the power of the king. Conway defined a *k-angel* to be a "king" that can "fly" in one move to any untouched square at distance at most k from his current position. He then asked whether the opponent—whom he figuratively calls *the devil*—can catch any angel of arbitrary power [1]. This problem remains unsolved for at least two decades now.

I took a very modest approached to this problem by considering an equivalent formulation where the angel may not fly but only run. In this setting, I investigated angels that run at small fractional speed greater than 1 and devised successful devil strategies for such slightly strengthened kings.

Current Work

My main efforts are still aimed at developing generalizations of a decomposition lemma for weak positional games, which is an essential ingredient for my theorem on rank-3 hypergraphs. A success in this direction might yield polynomial algorithms for a much larger class of hypergraphs. I also try to remove the almost-disjointness condition from my result. It should be possible to find a transformation that removes overlaps as a preprocessing step, so that the existing proof need not be modified.

Previous Work

I presented my results on the complexity of the computation of Boolean matrix roots [5] at *the Ninth International Computing and Combinatorics Conference (COCOON 2003)* in Big Sky, Montana, USA.

Last spring I worked on the *minimum enclosing ball problem* with Bernd Gärtner and Kaspar Fischer from ETH Zürich. I presented our paper [3] at last year's *European Symposium on Algorithms (ESA)* in Budapest.

Activities

Apart from the two conferences COCOON and ESA, I talked at the *Colloquium on Combinatorics* in Magdeburg, last November. In October I attended the CGC workshop and fall school at Neustrelitz.

Outlook

This winter, I am visiting Charles University Prague again. I will attend the block courses by Micha Sharir and Peter Cameron on geometric arrangements and permutation groups and also want to resume last year's work with Martin Loebl.

References

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[5] Martin Kutz. The complexity of Boolean matrix root computation. In T. Warnow and B. Zhu, editors, *COCOON 2003*, volume 2697 of *LNCS*, pages 212–221, 2003.

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