Semester Report WS03/04 of Oliver Klein

Name:	Oliver Klein
Supervisor:	Prof. Dr. Günter Rote
Field of Research:	Computational Geometry and Combinatorics
Topic:	Matching Shapes with a Reference Point
PhD Student	in the program since April 2003

Field of Research

Given two sets $A, B \in \mathcal{C}^2$, where $\mathcal{C}^2$ is the set of compact subsets of $\mathbb{R}^2$, one can be interested in how these sets resemble each other. One good measure of resemblance is the Hausdorff-Distance δ_H. This distance is defined as the smallest ε such that the Euclidian distance from every point of A to its nearest point of B is at most ε and vice versa. For measuring the resemblance of the two sets, one has to determine the minimal Hausdorff-Distance under a given set of transformations. For example, these transformations can be translations, rigid motions (translations and rotations) or even similarity transformations (rigid motions and scalings). Algorithms for determining the optimal transformation to minimize the Hausdorff-Distance are known in all three cases, but the run-time of these algorithms is not satisfying for most applications.

To decrease the run-time, the authors of [1] use reference points to get an approximation for the problem. A reference point is a mapping $r : \mathcal{C}^2 \to \mathbb{R}^2$ which fulfills two properties, namely

1. r is equivariant with respect to the set of transformations $\mathcal{T}$:

$$\forall T \in \mathcal{T}\ \forall A \in \mathcal{C}^2 :\ r(T(A)) = T(r(A))$$

2. r is a Lipschitz-continuous mapping in the following meaning:

$$\exists c \in \mathbb{R}_{>0}\ \forall A, B \in \mathcal{C}^2 : ||r(A) - r(B)|| \leq c\delta_H(A, B)$$

   In this context, c is called the quality of the reference point.

In [2] the Steiner point is shown to be a reference point with quality $\frac{4}{\pi}$. It is additionally shown, that this quality is optimal, which means that there cannot be any reference point with a smaller Lipschitz constant. This is shown by using strong functional-analytic tools and the axiom of choice. Therefore the proof is not constructive.

In [2] it is also shown, how an approximation algorithm using reference points with approximation ratio $1 + c$ with respect to translations can be developed. For other sets of transformations, this algorithm can be used in a natural way to reduce several degrees of freedom.

Summarizing the lower bound on the quality of a reference point of $\frac{4}{\pi}$ and the upper bound of the algorithm using reference points of $1 + c$ with respect to translations, where c is the quality of any reference point, it seems reasonable that there are sets $A_1, A_2, ..., A_n$ which cannot be matched in a way that

$$\forall i \neq j \in \{1, ..., n\} \ : \ \delta_H(A_i + t_i, A_j + t_j) \leq (1 + \frac{4}{\pi} - \varepsilon) \cdot \delta_H^{opt}(A, B),$$

where $\delta_H^{opt}(A, B)$ is the optimal Hausdorff-Distance which can be achieved under translations, $\varepsilon \in \mathbb{R}_{>0}$ is any constant and $t_i \in \mathbb{R}^2, i = 1, ..., n$ are translation vectors. Observe that under these assumptions the t_i can be interpreted as the reference points of the given sets.

In order to find compact convex sets in $\mathbb{R}^2$ which allow only an ε as small as possible in the above formula I've implemented two computer programs. The first of these uses linear programming and AMPL to calculate the minimal Hausdorff-Distance of two sets, which can be achieved under translation. The second one determines if it is possible to find translation vectors such that the above formula is fulfilled for a fixed ε, again using AMPL. By using these two programs it was possible for me to find sets A_i so that $\varepsilon \approx 1$. But I still haven't found sets, where $\varepsilon > 1$, although I don't think that $1+\frac{4}{\pi}-1 = \frac{4}{\pi}$ is the true lower bound for the problem. The next step will be to automate the approach and thereby getting a better lower bound.

Another problem we would like to solve is the extension of this problem to higher dimensions.

While reading a report by Gerald Weber, which is not yet published, concerning approximation algorithms using reference points with respect to the symmetric difference, my attention fell on a slightly stronger definition of regular reference points and some open problems attached to it. In this definition of regular reference points the set of allowed transformations to create the approximation is reduced. Therefore a general reference point need not be a regular reference point or it is a regular reference point, but with a worse approximation ratio. However, up to now there is no general reference point known, which is not a regular reference point with the same approximation ratio. It may be interesting to find such a reference point.

Activities

- Attended the lecture "Konvexe Geometrie" by Dr. Ivan Izmestiev at FU Berlin
- Attended the "Monday Lectures and Colloquia" of the Graduate Program
- Attended the "Mittagsseminar Theoretische Informatik" at FU Berlin. Presentation of the talks
  - "Detecting short cycles in directed graphs" based on [4]
  - "Output-Sensitive Algorithms for computing convex hulls" based on [3]
  - "The Centroid as a Reference Point for the Symmetric Difference in d Dimensions" based on [3]
- Attended the "Comprehensive Annual Workshop 2003", September 29 to October 1, 2003 in Neustrelitz. Presentation of the talk
  - "Lower bounds for shape matching with reference points", based on [2]
- Attended the "Fall School on Computational Geometry", October 2 to 4, 2003 in Neustrelitz.
- Attended the "Doktorandenworkshop des Instituts für Informatik der FU", October 24 to 25, 2003. Presentation of the talk
  - "Untere Schranken für den Vergleich geometrischer Formen mit Hilfe von Referenzpunkten", based on [2]
  - Extended Abstract was included in technical report B 03-17.
- Attended the "49. Workshop über Komplexitätstheorie, Datenstrukturen und effiziente Algorithmen (Theorietag)", November 21, 2003 at Heinrich-Heine-Universität Düsseldorf.

- Attending the “Blockcourse Berlin-Prague 2004”, January 26 to March 5, 2004 at Charles University, Prague
- Referee for ISAAC 2003
- Referee for SODA 2003

References

[1] H. Alt, B. Behrends, J. Blömer: 'Approximate matching of polygonal shapes', Proceedings 7th Annual Symposium on Computational Geometry, 1991, 186-193

[2] O. Aichholzer, H. Alt, G. Rote: 'Matching Shapes with a Reference Point', in International Journal of Computational Geometry and Applications, Volume 7, pages 349-363, August 1997

[3] T. M. Chan: 'Output-Sensitive Results on Convex Hulls, Extreme Points, and Related Problems', Discrete and Computational Geometry, Volume 16, pages 369-387, 1996

[4] R. Yuster, U. Zwick: 'Detecting short directed cycles using rectangular matrix multiplication and dynamic programming', Proceedings of the 15th Annual ACM-SIAM Symposium on Discrete Algorithms (2004), to appear