Semester Report WS02/03 of Ares Ribó Mor

Supervisor(s): Prof. Dr. Günter Rote
Field of Research: Geometry and Combinatorics
Topic: Locked and Unlocked Self-Touching Linkages; Map Foldability and Rigidity Theory

PhD Student at the program since February 1, 2002

Field of Research and Results

This semester I continued working on the topic of planar self-touching configurations. The context is the following. A *linkage* is a graph where edges are *rigid bars* with fixed length and vertices are flexible joints. A planar *configuration* of a linkage in $\mathbb{R}^2$ is a mapping of the vertices to points in $\mathbb{R}^2$ such that bars do not intersect. A linkage is *locked* if it can not be moved without crossings into some other configuration. In the plane, the combinatorial planar embedding of a linkage is specified because this cannot change by a motion that avoids crossings.

In a *self-touching configuration* bars are allowed to touch and even lie along each other, but not properly cross. The set of feasible motions can be described by linear equations and inequalities, which are stable at least in some neighbourhood of the self-touching configuration ([2]). A *δ-perturbation* of a self-touching configuration is a repositioning of the vertices within disks of radius δ consistent with the combinatorial embedding in $\mathbb{R}^2$.

Last semester I began to work on the following conjecture: for every self-touching configurations and for all $\delta > 0$, there is a δ-perturbation that is a configuration without bars crossing. We modelled the configuration by a system of linear inequalities, where the variables were the velocities of the vertices. We obtained a linear problem; its feasibility would give the existence of such a δ-perturbation. The idea was to show that the dual problem, which was threatened in terms of equilibrium stresses, had a bounded solution, and this would imply the feasibility of the primal problem. But this was very difficult to prove.

Motivated by this problem, and inspired by the idea of [1], we thought that probably it was a good idea to look at the three-dimensional lifting given by the *Maxwell-Cremona theorem* in order to find new conditions for the dual problem. The Maxwell-Cremona theorem, makes a one-to-one correspondence between the set equilibrium stresses of a planar configuration and the set of three-dimensional polyhedral terrain that project onto it.

But the *Maxwell-Cremona theorem* applies only to classical stresses, and does not cover self-touching configurations, thus we have been thinking of an extension of the theorem in this direction. Actually I am writing a generalisation of the Maxwell-Cremona theorem for self-touching configurations. In this new theorem, we give a one-to-one correspondence, between the set of stresses of a plane self-touching configuration and the set of three-dimensional polyhedral terrains with *jump discontinuities* that project onto it. This jump discontinuities are vertical facets given by the self-touching stresses.

With this interesting new tool, I pretend to attack our conjecture again, and I also want to apply it to study other problems regarding self-touching configurations, for example to see if a self-touching polygonal chain is always unlocked, or if an arbitrarily flattened tree can be locked.

During this months I also been working parallely on the problem of deciding the complexity of folding a $m \times n$ grid of paper with a mountain-valley assignment, i.e. with a given folding direction of the creases. This problem is studied since the end of the sixties, but there is no answer even for the case $2 \times n$. Studying local necessary conditions for flat foldability, I have written an algorithm, based on [4], to fold a $2 \times n$ grid with a mountain-valley assignment. Given a possible flat folding for the $2 \times n$-th grid, the algorithm finds all possible foldings (according to the mountain-valley assignment) for the grid $2 \times (n+1)$ in $O(n^2)$. But the complexity of the problem remains still open. I also want to continue the study of this nice problem.

Activities

- Attended the *Monday Lectures and Colloquia* of the Graduate Program. Presentation of the talk *Locked and Unlocked Self-Touching Configurations*, November 18, 2002

- Attended the *Mittagsseminar Theoretische Informatik* at Freie Universität Berlin. Presentation of the talks

  - *Locked and Unlocked Self-Touching Linkages*, September 24, 2002
  - *Folding a $2 \times n$ grid of paper*, January 16, 2003

- Lecture *Topologische Kombinatorik*, by Mark de Longueville at Freie Universität Berlin

- *Combinatorics Conference in Honour of Walter Deuber's 60th Birthday*, at Humboldt Universität zu Berlin, October 7–8, 2002
- Attended and presented a talk on *Locked and Unlocked Self-Touching Linkages* at the *Annual Workshop of the CGC Graduate Program*, Hiddensee, October 9–12, 2002
- Attended and presented a talk on *Self-Touching Linkages* at the First Meeting of the European Network *COMBSTRU I*, *Combinatorial Structure of Intractable Problems*, Prague, Czech Republic, October 24–27, 2002
- Learn& Work–shop *Algorithms, Structure, Randomness*, at Humboldt Universität zu Berlin, November 7–9, 2002
- Inauguration of the *DFG Research Center Mathematics for Key Technologies* at Technische Universität Berlin, November 11, 2002
- *Berliner Algorithmen-Tag*, at Konrad-Zuse-Zentrum für Informationstechnik, Berlin, February 7, 2003
- Subreferee for *STACS 2003*
- Subreferee for *STOC 2003*
- Lecture *Deutsch als Fremdsprache*, Mittelstufe II, at Volkhochschule Friedrischain–Kreuzberg

References

[1] R. Connelly, E. Demaine, G. Rote, *Straightening polygonal arcs and convexifying polygonal cycles*, to appear in Discrete & Computational Geometry.

[2] R. Connelly, E. Demaine, G. Rote, *Infinitesimally locked self-touching linkages with applications to locked trees*, "Physical Knots: Knotting, Linking, and Folding Geometric Objects in $\mathbb{R}^3$". Contemporary Mathematics 304, American Mathematical Society 2002, 287–311.

[3] H. Crapo, W. Whiteley, *Spaces of stresses, projections and parallel drawings for spherical polyhedra*, Contributions to Algebra and Geometry, Volume 35 (1994), No. 2, 259–281.

[4] W. F. Lunnon, *A map-folding problem*, Mathematics of Computation, Volume 22 (1968), 193–199.

[5] J. Richter-Gebert, *Realization spaces of polytopes*, chapter 13. Lecture Notes in Mathematics **1643**, Springer–Verlag Berlin Heidelberg 1996.