Semester Report WS02/03 of Katharina Langkau

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Field of Research:	Combinatorial Optimization
Topic:	Flows over Time with Flow-Dependent Transit Times
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Field of Research

During the last semester I have resumed my work on the topic of flows over time. The concept of flows over time was introduced by Ford and Fulkerson [3, 4]. It extends the standard network flow model by an extra time dimension.

We are considering a directed graph $G = (V, A)$, in which every arc $a \in A$ has a certain capacity u_a and a transit time τ_a. A flow over time f assigns every arc a and every point in time θ a certain flow rate value $f(a, \theta)$. The interpretation is the following: At time θ, flow f is entering arc a at rate $f(a, \theta)$. Once flow has entered arc a, it needs τ_a time units to reach the head node of arc a. Usually a time horizon T is given, after which no flow should be left in the network.

Motivated by applications in road traffic control, we have further generalized the notion of flows over time [5]. An important aspect of road traffic is that travel times depend on the current volume of traffic on the streets. Therefore, instead of assuming that all transit times are fixed, we allow τ_a to be a function of the incoming flow rate. In this setting we have considered the quickest multicommodity flow problem. Here, we are given k source-sink-pairs (s_i, t_i), $i = 1, \ldots, k$, with a certain demand d_i. The objective is to satisfy all demands as quickly as possible, i.e., we want to minimize the time horizon T.

Results

The method we have used is to expand the original graph according to the given transit time functions $(\tau_a)_{a \in A}$. More precisely, from the original graph G, we derive a so-called *bow graph* by introducing a certain number of copies of every arc. The copies of one particular arc represent all possible transit times on that arc. As a consequence, every arc in the bow graph has a fixed transit time and we can apply algorithms known in this much simpler setting.

In [1, 2], both a $(2 + \varepsilon)$-approximation algorithm and an FPTAS for the quickest multicommodity flow problem with fixed transit times is presented. The problem which arises when applying their algorithms is that the computed flow solutions in the bow graph do not automatically yield solutions in the original network. Together with Alexander Hall[1] and Martin Skutella[2], we could show that a refinement of their algorithms yields flow solutions in the bow graph which can be turned into feasible solutions in the original network. This approach gives rise to a fairly simple $(2 + \varepsilon)$-approximation algorithm and a more involved FPTAS for the quickest multicommodity flow problem with flow-dependent transit times.

Along with these algorithmic results, we were able to show certain NP-hardness results. For example, if intermediate storage of flow in vertices is not allowed, then the problem is NP-hard in the strong sense even if we restrict to the single source, single sink case.

Activities

- Talk on *Time-Expanded Graphs with Flow-Dependent Transit Times* at the 10th European Symposium on Algorithms (ESA), Rome, September 2002.
- Talk on *Flow-Dependent Transit Times: the Quickest Flow Problem* at the Annual Workshop of the European Graduate Program CGC, Hiddensee, October 2002.
- Talk on *Flow-Dependent Transit Times: the Quickest Multicommodity Flow Problem* at the Oberseminar Combinatorial Optimization and Graph Algorithms, TU Berlin, November 2002.

Literatur

[1] L. Fleischer and M. Skutella. *The Quickest Multicommodity Flow Problem*, in W. J. Cook and A. S. Schulz, editors, *Integer Programming and*

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[2] L. Fleischer, M. Skutella. *Minimum Cost Flows over Time without Intermediate Storage*, to appear in SODA 03, 2003.

[3] L. R. Ford and D. R. Fulkerson. *Constructing Maximal Dynamic Flows from Static Flows*, *Operations Research*, 6:419–433, 1958.

[4] L. R. Ford and D. R. Fulkerson. *Flows in Networks.* Princeton University Press, Princeton, NJ, 1962.

[5] E. Köhler, K. Langkau, M. Skutella. *Time-Expanded Graphs for Flow-Dependent Transit Times*, in Proceedings of the 10th Annual European Symposium on Algorithms (ESA), 2002, pp 599–611, Springer LNCS 2461.