Semester Report WS02/03 of Martin Kutz

Name:	Martin Kutz
Supervisor:	Prof. Dr. Martin Aigner
Field of Research:	Algorithmic Graph Theory, Combinatorial Game Theory
Topic:	Games on Hypergraphs
PhD Student	at the program since June 2001

Field of Research: Positional Games

In a *biased positional game* two players, called *Maker* and *Breaker*, alternately color the vertices of a hypergraph in their respective colors, Maker trying to create a monochromatic edge of his color—a Maker win. Breaker tries to prevent Maker from doing so and thus wins when at the end of the game, when all vertices are colored, no edge is monochromatic of Makers color. (He does not win by coloring one edge in his color, though; hence the term *biased*.) Such games have been studied, for example, in and [1] and [2].

Until now, the complexity of such games (that is, deciding whether a given position can be won) is unknown, while the unbiased version (the player who first completes one edge in his color wins) is known to be PSPACE-hard [3]. I suspected that unbiased games should be governed by computationally simpler principles and during the last semester I found out that for almost disjoint rank-3-bounded hypergraphs (each edge has at most three vertices and two different edges have at most one vertex in common) the winner of the game can be determined in polynomial time.

Current Work

Since October 2002, I am staying at Charles University Prague, with Prof. Martin Loebl. I am still working out the proof of the above result about games on rank-3-bounded hypergraphs. Its current version contains several unattractive case distinctions and deserves further investigation to understand better the "true" principles behind the theorem. We also try to get rid of the almost-disjointness condition in the statement. Further, we started to develop generalizations of a decomposition lemma, which is an essential ingredient for our proof. A success in this direction might yield polynomial algorithms for a much larger class of hypergraphs.

Side Effects

During the block course on randomized algorithms at the ETH Zürich, I got involved in the *minimum enclosing ball problem* from computational geometry [4]. Motivated by Bernd G"artners lectures on this topic, I implemented a novel algorithmic idea; and the first rough version of the resulting program showed surprising good performance. I would very much like to continue this work alongside my main topic, as far as the progress of my thesis allows.

Activities

- Attended the symposium "Diskrete Mathematik 2002", October 2 to 3, 2002 at TU Dresden.
- Attended the conference "Combinatorics — in Honour of Walter Deuber's 60th Birthday", October 7 to 8, 2002 at HU Berlin.
- Talk on "The Complexity of Boolean Matrix Root Computation" at the annual CGC workshop, October 9 to 12, 2002 on Hiddensee.
- Attended the bock courses "Randomized Algorithms" and "Approximation: Theory and Algorithms", October 21 to November 22, 2002 at ETH Zürich.
- Talk on "Weak Positional Games on 3-Uniform Hypergraphs" in the CGC colloquium, February 10, 2003 at TU Berlin.
- Since October 2002, abroad stay at Charles University Prague.

References

[1] Paus Erdös and J. L. Selfridge. On a combinatorial game. *Journal of Combinatorial Theory, Ser. A*, 14:298–301, 1973.

[2] A.W. Hales and R.I. Jewett. Regularity and positional games. *Transactions of the American Mathematical Society*, 106:222–229, 1963.

[3] Stephan Reisch. Gobang ist PSPACE-vollständig. *Acta Informatica*, 13:59–66, 1980.

[4] Emo Welzl. Smallest enclosing disks (balls and ellipsoids). In *New Results and New Trends in Computer Science*, volume 555 of *LNCS*, pages 359–370. 1991.