Semester Report WS02/03

of Dr. Mihyun Kang

Supervisor:	Prof. Dr. Hans Jürgen Prömel
Field of Research:	Random walks,
	random discrete structures,
	probabilistic methods
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Field of research and results

The connectivity of random graph processes

A random graph process is a Markov chain $\{G(n,M)\}_M$ of random graphs on vertex set $\{1,2,\cdots,n\}$, where $G(n,M+1)$ is obtained from $G(n,M)$ by adding a new edge according to some rule. The standard random graph process $\{G(n,M)\}_{M=0}^{\binom{n}{2}}$ introduced by Erdős and Rényi is an example of such a process, and it is shown that a.a.s. $G(n,M)$ becomes connected for $M = \frac{n}{2}(\log n + \Theta(1))$ (see [4] and [6]).

We consider a min-degree random multigraph process $\{G_{\min}(n,M)\}_{M\geq 0}$ where $G_{\min}(n,M+1)$ is obtained from $G_{\min}(n,M)$ by connecting a randomly chosen vertex of a minimum degree with another vertex of the graph (see [7]). We observe that the limit behaviour of $G_{\min}(n,M)$ is very different from the random graph process $\{G(n,M)\}_M^{\binom{n}{2}}$. We show that a.a.s. $G_{\min}(n,M)$ is connected when the minimum degree becomes three, and the probability that $G_{\min}(n,tn)$ is connected for $t \neq h_2 = \ln 2 + \ln(1+\ln 2)$ tends to a certain constant $\rho(t)$. The function $\rho(t)$ is continuous for all $t \neq h_2$. We show that for $M = (1+\delta)n$, $\delta > 0$, a.a.s. $G_{\min}(n,M)$ contains a giant component on at least $n/2$ vertices, but it seems to be a challenging problem to identify the moment when this giant component emerged. This is joint work with Y. Koh, T. Łuczak and S. Ree .

Generating labeled planar graphs uniformly at random

We derive a recursive counting formula for the number of labeled planar graphs with given number of vertices and edges, which yields an efficient uniform generation algorithm (see [3]). We use the block structure of a graph to uniquely decompose a 1-connected graph into 2-connected components: this technique has been used in [2] in order to generate outerplanar graphs uniformly at random. Then using a canonical decomposition of a 2-connected planar graph into its 3-connected components (see [11]), we lead the problem to count and generate 3-connected planar graphs. Due to the unique embedding property by Whitney, we can use the counting formula of rooted unlabeled 3-connected planar maps by Mullin and Schellenberg (see [8]), and the uniform generation algorithm of labeled 3-connected planar maps by Schaeffer (see [10]). This is joint work with M. Bodirsky, C. Gröpl.

Activities

- ADFOCS 2002: Advanced Course on Foundations of Computer Science (3rd Max-Planck Summer School), 9-13 September 2002, Saarbrücken.

- Fall School on Algorithms for Hard Problems, 23-27 September 2002, Schwarzenberg, Switzerland.

- Symposium Diskrete Mathematik 2002, 3-5 October 2002, Technische Universität Dresden.

- Gedenktagung Kombinatorik zu Ehren von Walter Deubers 60. Geburtstag, 7-8 October 2002, Humboldt Universität zu Berlin.

- Annual CGC workshop, 9-12 October 2002, Hiddensee, with a talk on *The Connectivity for the min-degree random graph process.*

- Visiting Adam Mickiewicz University in Poznań, 28 October 2002 - 11 April 2003. Seminarium ZMD with talks on *The mixing rate of a triangulation walk* and *Generating random planar graphs.*

- Learn & Workshop of the research group "Algorithms, Structure, Randomness", 7-10 November 2002, HU Berlin, with a talk on *Efficient generation of random discrete objects: triangulations and outerplanar graphs.*

- Oberseminar Theoretische Informatik, 3-4 December 2002, Technische Universität München, with a talk on *Generating random planar graphs.*

- 30.BAT, 7 February 2003, Konrad-Zuse-Zentrum für Informationstechnik Berlin, with a talk on *Uniform generation of planar graphs.*

References

[1] M. Bodirsky and M. Kang, Generating random outerplanar graphs, manuscript. ALICE03.

[2] M. Bodirsky, C. Gröpl and M. Kang, Generating labeled planar graphs uniformly at random, submmited.

[3] B. Bollobás, *Random Graphs*, Academic Press, London, 1985.

[4] S. Janson, T. Łuczak, A. Ruciński, *Random Graphs*, Wiley, New York, 2000.

[5] M. Kang, Y. Koh, T. Łuczak and S. Ree, The connectivity threshold for the min-degree random graph process, submmited.

[6] R.C. Mullin and P.J. Schellenberg, The enumeration of c-nets via quadrangulations, *J. Combinatorial Theory*, (4):259–276, 1968.

[7] G. Schaeffer, Random sampling of large planar maps and convex polyhedra, *In Proc. of the thirty-first annual ACM symposium on theory of computing (STOC'99)*, 760–769, Georgia, ACM press, 1999.

[8] T. Walsh, Counting unlabelled three-connected and homeomorphically irreducible two-connected graphs, *J. Combin. Theory B* **32** (1982), 12–32.