

Semester Report WS02/03 of Vanessa Kääb

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Field of Research:	Scheduling
Topic	Scheduling with AND/OR-networks
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Field of Research and Results

The last semester I considered the problem of scheduling with AND/OR-networks and additional machine constraints. In this setting, a set of jobs with strictly positive processing times has to be scheduled on one or several machines. The order of the jobs is restricted by AND/OR precedence constraints, represented by the given network. In an AND/OR-network the vertex set is composed of two disjoint sets, AND- and OR-nodes. The AND-nodes represent the jobs with the traditional precedence constraints: the processing of an AND-node has to be postponed until all its direct predecessors are finished. In contrast to this, the OR-nodes represent so-called waiting conditions. A waiting condition is a generalized precedence relation, where one job has to wait for the completion of at least one other job out of a pre-given set. Thus an OR-node can be scheduled as soon as one of its direct predecessors is finished. AND/OR precedence constraints are capable to model a much bigger variety of problems arising in the real world than the classical precedence constraints. Among others applications of scheduling with AND/OR-networks are resource constrained project scheduling, assembly sequencing problems, propositional games and public transportation problems.

Depending on the application various objective functions can be considered. We have been focusing on minimizing the makespan, that is the project completion time, the total completion time and the total weighted completion time. Especially in the area of assembly sequencing, the problem of scheduling jobs on one machine subject to AND/OR precedence constraints arises naturally. The precedence constraints model the different choices one has to disassemble or assemble a product, while the whole process has to be performed on one assembly line. Nevertheless the problem can also be useful to model certain situations in resource constrained project scheduling, where it might be computationally expensive to resolve the machine constraints by waiting conditions.

In [GL95], Gillies and Liu prove a $(2 - \frac{1}{m})$ -approximation algorithm for the problem of minimizing the makespan, where m is the number of machines. Additionally, they present NP-hardness results for a variety of problems with slightly different settings. We have been able to prove that a simple list scheduling algorithm is an n -approximation for the problem of minimizing the total completion time, where n is the number of jobs to be scheduled. Unfortunately, we can only present an example that is \sqrt{n} away from the optimum, thus it remains open, whether the list scheduling heuristic is actually an n -approximation or whether our analysis can be tightened to prove a better performance ratio. If we switch to the problem of minimizing the total weighted completion time, it turns out that this corresponds to a special assembly sequencing problem, which is at least as hard to approximate as the LABEL COVER problem. We presented a scheduling example forcing the same simple list scheduling algorithm used above a factor n away from the optimum, where again n is the number of jobs. In contrast to the unweighted case we were not able to prove a performance ratio of n for the list scheduling heuristic. Although it seems likely that the algorithm is an n -approximation, it remains open to be proved.

Recent work has been devoted to writing up my thesis, unfortunately not leaving much time for further research.

Activities

- *Monday Colloquia of the CGC Graduate Program.*
- “Local Search in AND/OR-Networks”, *Annual Workshop of the CGC Graduate Program*, Hiddensee, Germany, October 9–12, 2002.
- “Risikoanalyse in der Ressourcenbeschränkten Projektplanung”, *4th Siemens Workshop on Applied Discrete Optimization*, Nördlingen, Germany, Dezember 1–3, 2002.
- “Local Search in AND/OR-Networks”, *Monday Colloquium of the CGC Graduate Program*, Berlin, January 6, 2003.
- Support of a students talk in the *Block Seminar Graphen- und Netzwerkalgorithmen* by Rolf H. Möhring, Berlin, January 10–12, 2003.

Literatur

- [GL95] D. W. Gillies and J. W.-S. Liu. Scheduling Tasks with AND/OR Precedence Constraints. *SIAM Journal on Computing*, Volume 24, 797–810, 1995.