Semester Report WS02/03 of Andrzej Dudek

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Field of Research:	Discrete Mathematics
Topic:	Random graphs
PhD Student	at the program since September 2002

Field of Research

Given a graph H, a random maximal H-free graph is constructed by the following random greedy process. First assign to each edge of the complete graph on n vertices a birthtime which is uniformly distributed in $[0, 1]$. At time $p = 0$ start with the empty graph and increase p gradually. Each time a new edge is born, it is included in the graph if this does not create a copy of H. The question is then how many edges such a graph will have when $p = 1$. Denote the final graph by $M_n(H)$

The first result about the asymptotic structure of $M_n(H)$ is due to Ruciński and Wormald in 1995. They proved that if H is a star with $d+1$ edges then a.a.s. $M_n(H)$ is in fact an extremal H-free graph, i.e., it contains at most one vertex of degree $d-1$, and all other vertices will have degree d. The case when H is a triangle was investigated by Erdős, Suen, Winkler in 1995. If H is a triangle then a.a.s. $M_n(K_3)$ has close to $n^{3/2}$ edges. $M_n(K_3)$ is not extremal. The balanced complete bipartite graph has $\lfloor n^2/4 \rfloor$ edges.

We say that G is strictly 2-balanced if $v(G) \geq 3$ and $e(G) \geq 3$ and if for all proper subgraphs G' of G with $v(G') \geq 3$ we have $\frac{e(G)-1}{v(G)-2} > \frac{e(G')-1}{v(G')-2}$. Osthus and Taraz in 2001 proved that if H is strictly 2-balanced, where $v = v(H)$ and $e = e(H)$, then $e(M_n(H)) = n^{2-\frac{v-2}{e-1}+o(1)}$

The question is now what happens when the graph H is not strictly 2-balanced?

Results

I investigated that the number of edges of $M_n(H)$ is concentrated around more than one value, if H is the disjoint union of H_1 and H_2 where H_1 and H_2 are balanced. If $\frac{v(H_1)}{e(H_1)} > \frac{v(H_2)}{e(H_2)}$ then a.a.s. $e(M_n(H)) = e(M_n(H_2))$ but if $\frac{v(H_1)}{e(H_1)} = \frac{v(H_2)}{e(H_2)}$ then there exists a constant $\gamma = \gamma(H)$ so that a.a.s.

$Pr[e(M_n(H)) = e(M_n(H_1))] = \gamma$ and $Pr[e(M_n(H)) = e(M_n(H_2))] = 1 - \gamma$.

Activities

1. Combinatorics in honour of Walter Deuber's 60th Birthday, Humboldt-Universität zu Berlin, Germany, October 7 - 8, 2002
2. Annual Workshop on Combinatorics, Geometry, and Computation (with a talk on "Random maximal H-free graphs."), in Hiddensee, Germany, October 9 - 12, 2002
3. "Learn&work"-shop in Berlin, Germany, November 7 - 9, 2002
4. 30. Berliner Algorithmen Tag, Konrad-Zuse-Zentrum für Informationstechnik, Berlin, February 7, 2003
5. Lectures and colloquia of the European graduate program "Combinatorics, Geometry, and Computation" (with a talk on "Random maximal H-free graphs-special cases"), Winter Term, 2002
6. Seminar "Algorithms and Complexities" at the HU, Winter Term, 2002
7. German Course at the Volkshochschule *"Mittelstufe 2"*, Berlin, Winter Term, 2002
8. German Course at the HU *"Mittelstufe 2"*, Berlin, Winter Term, 2002
9. Spanish Course at the HU *"El lenguaje de la prensa"*, Berlin, Winter Term, 2002
10. Spanish Course at the Instituto Cervantes *"Oberstufe"*, Berlin, since 20 Janury

Preview

I want to investigate a general case when H is a arbitraly graph. I think that if $\max_{H' \subseteq H} \frac{e(H')}{v(H')}$ is for a unique H' then $e(M_n(H)) = e(M_n(H'))$.

References

[1] N. Alon and J.H. Spencer, *The Probabilistic Method*, Wiley-Interscience, New York, 1992.

[2] S. Janson, T. Łuczak, and A. Ruciński *Random Graphs*, Wiley-Interscience. New York, 2000.

[3] D. Osthus, H.J. Prömel, and A. Taraz, *On the evolution of triangle-free graphs*, 1999.

[4] D. Osthus, A. Taraz, *Random maximal H free graphs*, RSA 18 (2001), 61-82.

[5] A. Ruciński and N.C. Wormald, *Random graph processes with degree restrictions*, Combin. Probab. Comput. 1 (1992), 169-180.