

# Semester Report WS02 of Manuel Bodirsky

Name: Manuel Bodirsky  
Supervisor: Prof. Dr. Hans Jürgen Prömel  
Topics: Graph Algorithms for Constraint Satisfaction;  
Generation of Random Labeled and Unlabeled  
Outerplanar and Planar Graphs;  
PhD Student At the program since April 2001

## Fields of Research and Results

The topic of my PhD thesis are efficient algorithms and complexity results for combinatorial constraint satisfaction problems. In particular, I'm interested in various *tree description languages*. Like Allen's interval constraints, they can not be formulated in the well-studied framework called CSP. Problems in CSP are given by a finite relational structure  $T$  (the *template*); we have to decide for a given relational structure  $S$  whether there exists a homomorphism from  $S$  to  $T$ . But still, the mentioned tree description languages and Allen's interval constraints can be described using the notion of homomorphism: If we drop the condition of  $T$  being finite, we can formulate the problems using an appropriate *countable homogenous universal* template  $T$ . For pure dominance constraints for instance, the homogeneous universal structure is *the unbounded infinitely branching and dense* directed tree.

Countable homogeneous relational structures are not yet classified, despite considerable effort. However, all different types for graphs [1], tournaments and digraphs [2] are known. Model theorists are interested in homogeneous structures, as these structures have many interesting properties, for instance they often allow quantifier elimination.

To study the expressive power of a constraint language, we want to characterize the relations that can be *expressed* by other relations, i.e. the relations definable over the homogeneous structure  $T$  by existential conjunctive (also called *primitive positive*) first order formulas. The set of all these relations we denote by  $\langle T \rangle$ . Combining quantifier elimination, the known Galois-correspondence techniques for Inv-Pol and a compactness argument we can show that these relations can be characterized by the polymorphism of the homogeneous template  $T$  (i.e., homomorphisms from  $T^k$  to  $T$ ). One question that I am interested in is whether it is possible to interpret the existence of the various graph algorithms we found for tree description languages (men-

tioned in my earlier reports) as the existence of certain polymorphisms of the corresponding homogenous structures.

**Generation of Random Structures.** We also continued our work on efficient generation of planar and outerplanar graphs uniformly at random. For labeled planar graphs, I presented this joint work with Mihyun Kang and Clemens Groepl at the CGC meeting in Hiddensee. For labeled and unlabeled outerplanar graphs, I presented our results at the first workshop on Algorithms for Listing, Counting, and Enumeration (ALICE03) associated to this years Symposium on Discrete Algorithms (SODA). Currently we try to apply our generation method via recursive counting procedures to generate also unlabeled random planar graphs, using the structure of the triconnected components [3] and Tutte's census of planar maps [4].

This semester I spent in Prague at the Charles University. I very much appreciated, and want to thank for, the friendly, open and dense atmosphere at ITI and DIMATIA.

## Activities

Attended lectures, schools, workshops, conferences:

- ADFOCS: 3rd Max-Planck Advanced Course on the Foundations of Computer Science, Saarbrücken, September 9 - 13, 2002.
- CGC Fall School *Algorithms for Hard Problems*, Bildungszentrumm Matt, Schwarzenberg, Switzerland, September 23-27, 2002.
- Diskrete Mathematik 2002, Dresden, October 3-4, 2002.
- Combinatorics in honour of Walter Deuber's 60th Birthday, Humboldt-Universität zu Berlin, October 7-8, 2002.
- CGC workshop Hiddensee, October 9-12, 2002.
- Learn- and Workshop Humboldt-Universitt zu Berlin, 7th to 9th November 2002.
- 14th Annual ACM-SIAM Symposium on Discrete Algorithms, Baltimore, MD, USA, January 12-14, 2003.

Talks:

- *Surjective Homomorphism Problems*, at Homonolo02 (Workshop on graph homomorphisms in Nova Louka), October 4, 2002.
- *Generating Random Outerplanar Graphs*, at ALICE03 (1st Workshop on Algorithms for Listing, Counting, and Enumeration), Baltimore, January 12-14, 2003.

## Preview

- 65th Workshop on General Algebra, 18th Conference for Young Algebraists, in Potsdam, March 21-23, 2003.
- Novi Sad Algebraic Conference (NSAC '03), Novi Sad, August 25-28, 2003.
- Organizing the seminar *the strange logic of random graphs*, (jointly with Mihyun Kang).

## Literatur

- [1] A.H.Lachlan and R. Woodrow. Countable ultrahomogeneous undirected graphs. *Trans. Amer. Math. Soc.*, 262:51–94, 1980.
- [2] G. Cherlin. *The Classification of Countable Homogeneous Directed Graphs and Countable Homogeneous  $n$ -Tournaments*, volume 131. AMS Memoir, 1998.
- [3] J. Hopcroft and R. E. Tarjan. Dividing a graph into triconnected components. *SIAM J. Comput.*, 2:135–158, 1973.
- [4] W. Tutte. A census of planar maps. *Canad. J. Math.*, 15:249–271, 1963.